Physics Letters A ●●● (●●●●) ●●●–●●●

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Scattering of electromagnetic wave by vortex flow

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ARTICLE INFO

Article history:
Received 9 November 2016
Received in revised form 20 February 2017
Accepted 23 February 2017
Available online xxxx
Communicated by V.A. Markel

Keywords:
Vortex flow
Differential cross-section
Scattering characteristics
Frequency selectivity

ABSTRACT

In this paper, the scattering behaviour of an electromagnetic wave by vortex flow is studied in detail by solving the first-order (in v/c) Maxwell's equation in the cylindrical coordinate system (r, φ, z) and the general solutions are obtained. From these solutions, the differential cross-section of the vortex flow is calculated and the electromagnetic scattering characteristics of the vortex flow are discussed. The dependence of differential cross-section on the velocity profile and the radius of the vortex flow is investigated independently. Besides, by considering the dependence of scattering characteristics on the frequency of an incident wave we conclude that the vortex flow has frequency selectivity.
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1. Introduction

The electrodynamics in moving media has made significant progress since the birth of Maxwell's equations and Minkowski's constitutive relations, and these relationships are very useful in explaining the electromagnetic phenomena related to the moving media. Extensive research on the propagation and scattering of electromagnetic waves in the moving media has been carried out and several important findings have been reported such as the frequency shift of a reflected wave by a moving mirror [1,2], the Aharonov–Bohm effect [3] for light by moving media, the generation of negative refraction [4] by moving media, and the magneto-electric effect of a moving medium [5]. More recently, a new theory to achieve birefringence in the time-dependent moving media has been given [7,8]. Such findings rise interest in the electrodynamics theory of the moving media and lead physicists to discuss more interesting situations, one of these is the rotating medium, specifically the case of a rotating cylinder. The scattering problem of static cylinder is well established and the analytical solutions are given in many textbooks on the electromagnetic scattering theory, however, the problem becomes complex if the cylinder is rotating. In such a case when the cylinder is rotating very fast, the effects of velocity cannot be ignored. Some authors have studied the scattering problem of rotating bodies [9–13], besides, the solutions for a rotating dielectric cylinder with a certain circular mechanical frequency are also given [14,16,40]. Although the electrodynamics of rotating systems has been much discussed, it has a long tradition in providing controversy. Gerald and coauthors questioned the rationality of the theoretical description of Wilson–Wilson experiment which applied the theoretical result of translational media to rotating media [31]. Since then, several articles have been reported which discussed the Wilson–Wilson experiment and presented the analytical solutions that describe the electrodynamics of rotating media [32–37]. The Abraham–Minkowski debate also provides numbers of articles about the rotating media [38–42]. Most of the references given herein discuss the electromagnetic field in a rotating medium, however, we are interested in to observe the influence of a rotating medium on the scattering field outside the rotating medium. It will be helpful in acquiring useful information of an unknown vortex through its scattering characteristics.

In this paper, a relatively complex model of a rotating dielectric cylinder is studied. The scattering problem of an infinitely long dielectric vortex flow with the velocity profile $\mathbf{v} = \Gamma/r\hat{\mathbf{e}}_\varphi$ [15] is discussed where the velocity profile of the vortex flow satisfies Euler's equation for an ideal incompressible fluid i.e., $dv/v + dr/r = 0$. Besides, the velocity field is divergence and curl free i.e., $\nabla \cdot \mathbf{v} = 0$ and $\nabla \times \mathbf{v} = 0$. The electrodynamics of such a moving medium has been studied by many authors and many important findings have been reported. D. Censor has discussed the first-order propagation in the waveguide filled with the vortex flow [17], U. Leonhardt has studied the Aharonov–Bohm effect for light [6], and the optical black hole effect near the vortex core for the vortex flow has also been reported [18]. The scattering of an electromagnetic wave by vortex flow in a rotating dielectric medium has not been studied before, therefore, we discuss such a scattering behaviour in detail in our work. We have derived the essential equations for an electromagnetic field that satisfies vortex flow and further, discussed the general solutions. We have also studied the scatter-

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http://dx.doi.org/10.1016/j.physleta.2017.02.041
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ing characteristics of the vortex flow and deduced very interesting conclusions.

The paper is organized as follows. In Section 2, the electromagnetic field equations for the vortex flow are derived by using the Maxwell's equations. The general solutions of these equations are given in Section 3. In Section 4, the scattering problem for the vortex flow is discussed. The scattering characteristics of the vortex flow are discussed in detail in the Section 5 and the last Section 6 concludes our work.

2. Essential equations

We start with the well known Maxwell's equations which for a source free medium can be written as

$$\begin{aligned} &\nabla\cdot\mathbf{D}=0, \quad \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\\ &\nabla\cdot\mathbf{B}=0, \quad \nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}. \end{aligned} \tag{1}$$

The constitutive relations between these fields for a moving medium in the laboratory system can be approximated by Minkowski's relations of the first-order (in v/c) [19] when we have $v<c$. Therefore, we can write

$$\begin{aligned} &\mathbf{D}=\epsilon\mathbf{E}+\left(\frac{n^2-1}{c^2}\right)\mathbf{v}\times\mathbf{H},\\ &\mathbf{B}=\mu\mathbf{H}-\left(\frac{n^2-1}{c^2}\right)\mathbf{v}\times\mathbf{E}, \end{aligned} \tag{2}$$

where ϵ and μ denote the permittivity and the permeability of the medium respectively. In our case, both the permittivity and permeability of the medium are considered as constants. In the same set of equations given earlier, n is the refractive index of the medium, $\mathbf{v}$ is the velocity of the vortex flow, and c is the speed of light in vacuum.

To find the solutions for a monochromatic electromagnetic wave, we assume the field in harmonic form, and write

$$\begin{aligned} &\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})\exp(i\omega t),\\ &\mathbf{H}(\mathbf{r},t)=\mathbf{H}(\mathbf{r})\exp(i\omega t). \end{aligned} \tag{3}$$

Further solving Eqs. (2) and (3), and then substituting the result into the curl equations given in Eq. (1), we arrive at the two equations. The resulting equations are given by

$$\begin{aligned} &\nabla\times\mathbf{E}=-i\omega\mu\mathbf{H}+i\omega\left(\frac{n^2-1}{c^2}\right)\mathbf{v}\times\mathbf{E},\\ &\nabla\times\mathbf{H}=i\omega\epsilon\mathbf{E}+i\omega\left(\frac{n^2-1}{c^2}\right)\mathbf{v}\times\mathbf{H}. \end{aligned} \tag{4}$$

For the divergence relations in the Maxwell's equations, it's easy to prove that for the electromagnetic field of harmonic form like Eq. (3), the curl relations in the Maxwell's equations satisfy the divergence relations. In fact, the divergence relations act just like initial conditions, once the initial values of the divergences are given, the values of any other moment will remain the same.

Next, we consider the vortex flow in an incompressible liquid with velocity represented in the cylindrical coordinate system as

$$\mathbf{v}=\frac{\Gamma}{r}\hat{\mathbf{e}}_\varphi, \tag{5}$$

where the parameter Γ is a constant independent of the coordinates with dimension $[\mathrm{L}^2\mathrm{T}^{-1}]$. It is obvious that the velocity is centrosymmetric about the z-axis, therefore, the scattering problem can be simplified to a two-dimensional problem if the direction of the incident wave is perpendicular to the symmetry axis.

Now, consider a case when the spatial part of the electromagnetic field depends only on r and φ. By expanding the set of Eq. (4) using the cylindrical coordinates (r,φ,z) and further using Eq. (5) into it, the set of Eq. (4) transforms to

$$\hat{\mathbf{e}}_r:\quad H_r=\frac{1}{i\omega\mu}\left(\frac{i\alpha}{r}E_z-\frac{1}{r}\frac{\partial E_z}{\partial\varphi}\right), \tag{6}$$

$$\hat{\mathbf{e}}_\varphi:\quad H_\varphi=\frac{1}{i\omega\mu}\frac{\partial E_z}{\partial r}, \tag{7}$$

$$\hat{\mathbf{e}}_z:\quad \frac{\partial E_\varphi}{\partial r}+\frac{E_\varphi}{r}-\frac{1}{r}\frac{\partial E_r}{\partial\varphi}=-\frac{i\alpha}{r}E_r-i\omega\mu H_z, \tag{8}$$

and

$$\hat{\mathbf{e}}_r:\quad E_r=\frac{1}{i\omega\epsilon}\left(-\frac{i\alpha}{r}H_z+\frac{1}{r}\frac{\partial H_z}{\partial\varphi}\right), \tag{9}$$

$$\hat{\mathbf{e}}_\varphi:\quad E_\varphi=-\frac{1}{i\omega\epsilon}\frac{\partial H_z}{\partial r}, \tag{10}$$

$$\hat{\mathbf{e}}_z:\quad \frac{\partial H_\varphi}{\partial r}+\frac{H_\varphi}{r}-\frac{1}{r}\frac{\partial H_r}{\partial\varphi}=-\frac{i\alpha}{r}H_r+i\omega\epsilon E_z. \tag{11}$$

In the equations given above, $\alpha=\frac{n^2-1}{c^2}\omega\Gamma$ is a dimensionless quantity and obviously for $\alpha=0$, i.e., $\mathbf{v}=0$, these equations reduce back to the usual case when there is no vortex flow. It is evident from Eqs. (6) and (7) that if the electric field component E_z is known, it is easy to determine the magnetic field components H_r and H_φ. Similarly, from Eqs. (9) and (10), it is clear that if the magnetic field component H_z is known, the electric field components E_r and E_φ can be determined. Thus, we only need to find the equations that satisfy E_z and H_z.

On substituting Eqs. (6)–(7) and Eqs. (9)–(10) into Eqs. (11) and (8) respectively, we arrive at the second order differential equations

$$\frac{\partial^2E_z}{\partial r^2}+\frac{1}{r}\frac{\partial E_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2E_z}{\partial\varphi^2}-\frac{2i\alpha}{r^2}\frac{\partial E_z}{\partial\varphi}+\left(k^2-\frac{\alpha^2}{r^2}\right)E_z=0, \tag{12}$$

and

$$\frac{\partial^2H_z}{\partial r^2}+\frac{1}{r}\frac{\partial H_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2H_z}{\partial\varphi^2}-\frac{2i\alpha}{r^2}\frac{\partial H_z}{\partial\varphi}+\left(k^2-\frac{\alpha^2}{r^2}\right)H_z=0, \tag{13}$$

where $k=\sqrt{\mu\epsilon}\,\omega$ is the wave number in static medium. We notice from Eq. (12) and Eq. (13) that the field components E_z and H_z satisfy the same differential equation which implies that if one of the two equations is solved, the other is also known by the same method.

By comparing Eq. (12) with the usual Helmholtz equation in the cylindrical coordinates, we observe two new terms, the first order derivative term $(-\frac{2i\alpha}{r^2}\frac{\partial E_z}{\partial\varphi})$ and the zero order derivative term $(-\frac{\alpha^2}{r^2}E_z)$, both are vortex dependent. If we set $\alpha=0$, the Eq. (12) reduces back to the well known Helmholtz equation. In the next section, we discuss the analytical solutions for the field component E_z by using the variable separation method.

3. General solution with variable separation

We use the separation of variables method to solve the prior given Eq. (12). In doing so, the product form for the field component E_z can be written as

$$E_z(r,\varphi)=R(r)\,\Phi(\varphi). \tag{14}$$

By substituting Eq. (14) into Eq. (12) and then multiplying the resulting equation with $r^2/R\Phi$, we arrive at

$$\frac{r^2}{R}\frac{\partial^2R}{\partial r^2}+\frac{r}{R}\frac{\partial R}{\partial r}+k^2r^2+\frac{1}{\Phi}\frac{\partial^2\Phi}{\partial\varphi^2}-\frac{2i\alpha}{\Phi}\frac{\partial\Phi}{\partial\varphi}-\alpha^2=0. \tag{15}$$

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