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Experimental study on slow flexural waves around the defect modes in a phononic crystal beam using fiber Bragg gratings

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A B S T R A C T

This work experimentally studies influences of the point defect modes on the group velocity of flexural waves in a phononic crystal Timoshenko beam. Using the transfer matrix method with a supercell technique, the band structures and the group velocities around the defect modes are theoretically obtained. Particularly, to demonstrate the existence of the localized defect modes inside the band gaps, a high-sensitivity fiber Bragg grating sensing system is set up and the displacement transmittance is measured. Slow propagation of flexural waves via defect coupling in the phononic crystal beam is then experimentally demonstrated with Hanning windowed tone burst excitations.

1. Introduction

Phononic crystals (PCs) have received considerable attention for their potential applications in various wave-manipulation devices. PCs are artificial structures consisting of alternating segments with large contrast in material or geometric properties [1]. Due to Bragg scattering or local resonance, PCs have frequency band gaps in which acoustic/elastic waves are strongly attenuated in a certain direction [2]. The existence of the band gaps in PCs is of interest to many researchers and has led to various practical applications such as vibration control, sound insulation, frequency filters, and so on [3–5].

In addition to the band gap phenomena, confinement of acoustic/elastic waves in localized modes is possible through the introduction of crystal defects. A point defect acts as a microcavity which locally disturbs the crystal periodicity and generates the defect modes to confine the acoustic/elastic waves inside the band gaps [6,7]. When line defects are created in a PC, waves are guided along the defects and thus they can be used as an efficient waveguide. Despite the fact that various properties or devices have been proposed relating to the defect modes [6–10], less attention has been paid to PC beam structures in consideration of the defect modes and their associated flexural wave propagations.

Band structures of flexural waves in a PC with a point defect can be theoretically studied using the plane wave expansion (PWE) method with a supercell technique [11,12]. PC plates with line defects have also been studied using the finite difference time domain (FDTD) method [13,14]. Recently, Hou et al. investigated the band structures of surface acoustic waves on nanostructured PCs with defects based on Brillouin light scattering and finite element simulations [15]. For sound waves in phononic crystals, Robertson et al. studied slow group velocity propagation of sound due to defect coupling in an acoustic diameter-modulated waveguide [16]. For light in photonic crystals, the so-called slow light caused by the defect modes provides a way to achieve time delay for optical systems [17]. However, the group velocity of flexural waves around the defect modes in PC beams is seldom theoretically or experimentally studied.

In this letter, propagations and transmission properties of the defect modes of flexural waves in a PC Timoshenko beam (hereafter simply referred as a PC beam) with a point defect are investigated. We specifically focus on experimental demonstration of the existence of the defect modes inside the band gaps and their influences on the group velocity of flexural waves. Band structures are obtained using the transfer matrix method (TMM) with a supercell technique. In addition, the group velocity of the defect modes is calculated from the dispersion curves. Particularly, two point-wise fiber Bragg grating (FBG) sensors are set up to measure the displacement transmittance at the two ends of the PC beam and to verify the existence of the defect modes. The FBG sensing system

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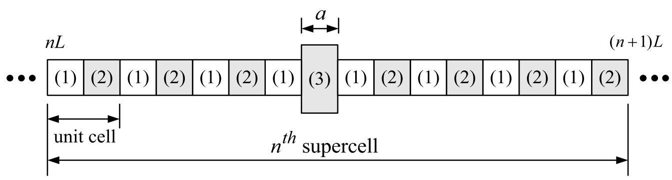

Fig. 1. Schematic diagram of the supercell of the PC beam.

is also employed to measure transient flexural wave propagations excited by N-cycle tone bursts for calculations of the group velocities. To the best of the authors' knowledge, this is the first research in phononic crystals using a high-sensitivity point-wise FBG displacement sensing system. Due to its capability of performing simultaneous in-plane or out-of-plane point-wise displacement measurements, this research opens the possibility of employing the FBGs for experimental demonstrations of various phenomena (e.g., negative refraction, focusing, and so on) in periodic structures such as elastic phononic crystals or metamaterials.

2. Transfer matrix method with a supercell technique

A transfer matrix method (TMM) is an efficient analytical method very suitable for calculation of dispersive curves or transmission properties of one-dimensional periodic structures such as rods or beams. In this section, the TMM with a supercell technique is adopted to analyze the defect modes in an infinite PC beam. A supercell is a large cell containing alternating unit cells with a defect in the center. Fig. 1 illustrates the nth supercell of the infinite PC beam. The supercell consists of 8 unit cells in which each unit cell is built of two segments with different elastic constants and the crystal defect is introduced by varying the height of the middle segment (i.e., labeled as (3) in Fig. 1).

Before applying the TMM to the PC beam, the governing equation of the free bending vibration of a homogeneous Timoshenko beam segment with a constant cross-section is considered [18,19]:

$$\rho S\frac{\partial^2 w(x,t)}{\partial t^2}+EI\frac{\partial^4 w(x,t)}{\partial x^4}-\rho I\left(1+\frac{E}{\kappa G}\right)\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2}+\frac{\rho^2 I}{\kappa G}\frac{\partial^4 w(x,t)}{\partial t^4}=0, \tag{1}$$

where ρ, S, E, G, I, and κ respectively represents the density, the cross-sectional area, the Young's modulus, the shear modulus, the cross-sectional area moment of inertia, and the Timoshenko shear coefficient. For a steady-state vibration response, the amplitude of the transverse deflection $w(x,t)=\phi(x)e^{i\omega t}$ satisfies:

$$\phi^{(4)}(x)-\alpha\phi^{(2)}(x)-\beta\phi(x)=0, \tag{2}$$

where

$$\alpha=-\frac{\rho\omega^2}{E}-\frac{\rho\omega^2}{\kappa G} \tag{3a}$$

and

$$\beta=\frac{\rho S\omega^2}{EI}-\frac{\rho^2\omega^4}{E\kappa G}. \tag{3b}$$

By solving the corresponding characteristic equations, the general solution for Eq. (1) can be expressed as a linear combination of cosine, sine, hyperbolic cosine, and hyperbolic sine functions as follows:

$$\phi(x)=A\cos(\lambda_1 x)+B\sin(\lambda_1 x)+C\cosh(\lambda_2 x)+D\sinh(\lambda_2 x), \tag{4}$$

where

$$\lambda_1=\sqrt{\sqrt{\beta+\frac{\alpha^2}{4}}-\frac{\alpha}{2}} \tag{5a}$$

and

$$\lambda_2=\sqrt{\sqrt{\beta+\frac{\alpha^2}{4}}+\frac{\alpha}{2}}. \tag{5b}$$

Next, we consider the supercell of the infinite PC Timoshenko beam with a point crystal defect illustrated in Fig. 1. The amplitude of the flexural deflection of the jth segment in the nth supercell can be described as:

$$\phi_n^j(x_j)=A_n^j\cos(\lambda_1^j x_j)+B_n^j\sin(\lambda_1^j x_j)+C_n^j\cosh(\lambda_2^j x_j)+D_n^j\sinh(\lambda_2^j x_j), \tag{6}$$

where $x_j=x-nb-(j-1)a$ and $nb+(j-1)a\le x\le nb+ja$, $j=1,2,3,\ldots,r$. The four interfacial continuity conditions of the transverse displacement, the angle of rotation, the bending moment, and the shear force between jth and $(j-1)$th segment can be written as:

$$\phi_n^j(0)=\phi_n^{j-1}(a) \tag{7a}$$

$$\phi_n'^{\,j}(0)=\phi_n'^{\,j-1}(a) \tag{7b}$$

$$EI\phi_n''^{\,j}(0)=EI\phi_n''^{\,j-1}(a) \tag{7c}$$

$$EI\phi_n'''^{\,j}(0)=EI\phi_n'''^{\,j-1}(a). \tag{7d}$$

Eqs. (7) can be further expressed in a matrix form:

$$\mathbf{K}_j\boldsymbol{\Psi}_n^j=\mathbf{H}_{j-1}\boldsymbol{\Psi}_n^{j-1}, \tag{8}$$

where

$$\boldsymbol{\Psi}_n^j=\left[A_n^j B_n^j C_n^j D_n^j\right]^T, \tag{9a}$$

$$\mathbf{K}_j=\begin{bmatrix}1&0&1&0\\0&\lambda_1^j&0&\lambda_2^j\\-(\lambda_1^j)^2&0&(\lambda_2^j)^2&0\\0&-(\lambda_1^j)^3&0&(\lambda_2^j)^3\end{bmatrix}, \tag{9b}$$

and

$$\mathbf{H}_{j-1}=\begin{bmatrix}\cos(\lambda_1^{j-1}a)&\sin(\lambda_1^{j-1}a)&\cosh(\lambda_2^{j-1}a)&\sinh(\lambda_2^{j-1}a)\\-\lambda_1^{j-1}\sin(\lambda_1^{j-1}a)&\lambda_1^{j-1}\cos(\lambda_1^{j-1}a)&\lambda_2^{j-1}\sinh(\lambda_2^{j-1}a)&\lambda_2^{j-1}\cosh(\lambda_2^{j-1}a)\\-(\lambda_1^{j-1})^2\cos(\lambda_1 a)&-(\lambda_1^{j-1})^2\sin(\lambda_1 a)&(\lambda_2^{j-1})^2\cosh(\lambda_2^{j-1}a)&(\lambda_2^{j-1})^2\sinh(\lambda_2^{j-1}a)\\(\lambda_1^{j-1})^3\sin(\lambda_1 a)&-(\lambda_1^{j-1})^3\cos(\lambda_1 a)&(\lambda_2^{j-1})^3\sinh(\lambda_2^{j-1}a)&(\lambda_2^{j-1})^3\cosh(\lambda_2^{j-1}a)\end{bmatrix}. \tag{9c}$$

Then, the relationship between the adjacent cells can be established by a transfer matrix as:

$$\boldsymbol{\Psi}_{n+1}^1=\mathbf{T}\boldsymbol{\Psi}_n^1, \tag{10}$$

where $\mathbf{T}=\mathbf{K}_1^{-1}\mathbf{H}_r\mathbf{K}_r^{-1}\mathbf{H}_{r-1}\cdots\mathbf{K}_2^{-1}\mathbf{H}_1$. Since now the flexural wave is propagated in an infinite periodic beam, the well-known Bloch theorem is satisfied and can be applied between the state vectors of the adjacent supercells as:

$$\boldsymbol{\Psi}_{n+1}^1=e^{iqL}\boldsymbol{\Psi}_n^1, \tag{11}$$

where q is the wavenumber. By solving the corresponding eigenvalue problem obtained by subtracting Eq. (11) from Eq. (10):

$$\left|\mathbf{T}-e^{iqL}\mathbf{I}\right|=0, \tag{12}$$