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# Topological charge identification of partially coherent light diffracted by a triangular aperture

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ABSTRACT

By using a triangular shaped aperture it was shown recently that the topological charge of a coherent optical beam could be measured. As the diffraction pattern generated by any setup depends exclusively on the coherence properties of the optical field, we have extended this method by including partially coherent light into the system.



## 1. Introduction

Since the discovery of phase singularities (wave dislocations, optical vortices) in 1974 a new branch of optical research, now named singular optics, started to be extensively investigated by numerous research groups [1,2]. For a given scalar solution of the wave equation there may be lines in space where the amplitude of the wave vanishes and therefore its phase becomes ill-defined. These are lines in three dimensions because the definition for these singular points requires the intersection of two (in general, generic) surfaces: the real and imaginary parts of the complex scalar solution must both vanish. If one is dealing with high symmetrical physical systems (such as laser modes) the singular regions may form surfaces in three dimensions rather than lines. One possible way to characterize these singular phase points is to observe how much the phase increases around a specific singularity from an arbitrary predefined fixed orientation sense and to divide this number by $2\pi$. The value obtained is called the topological charge and it may assume $\pm 1$ values for generical wavefields although it may assume any integral value for nongeneric solutions [3]. Eighteen years later, L. Allen et al. correctly associated an orbital contribution to the angular momentum of a light beam [4]. It turned out that this contribution was due to an $\exp(im\phi)$ phase term where $m$ is an integer and $\phi$ the polar angle in cylindrical coordinates, leading to a screw phase singularity, and this is the main reason why people usually associate phase singularities with orbital angular momentum. It should be remarked that this association is not entirely correct but it does make sense for some specific solutions such as Laguerre–Gaussian beams [5,6]. Although the total angular momentum of a light beam is not directly associated with phase singular points, the beam angular momentum density plays a dynamical role in the occurrence of such singularities. Optical beams with screw phase singularities may be used to rotate particles and the resulting topological charge has a direct connection with this interaction [7,8]. Therefore, topological charge measurement is an important issue and it was addressed by several authors (see for example, [9]). In particular, an ingenious diffraction-based method has been proposed in which the counting of maxima in a diffraction pattern determines both sign and modulus of the topological charge of an incident beam, possessing a nongeneric screw phase singularity, after it goes through a triangular aperture [10]. However, in experiments like that where the detailed structure of diffraction is important, coherence effects should influence in a decisive manner the results in such a way that by ignoring the statistical aspects of the incident field, one should expect considerable divergence between theory and experiment. Furthermore, to acquire knowledge on correlation singularities is essentially important in a wide range of phenomena such as imaging science, studies on the propagation of vortex fields through atmospheric turbulence, or even in astrophysics where partially coherent light is often present. In particular, the results reported here can affect severely the propagation of optical vortices through atmospheric turbulence and consequent the

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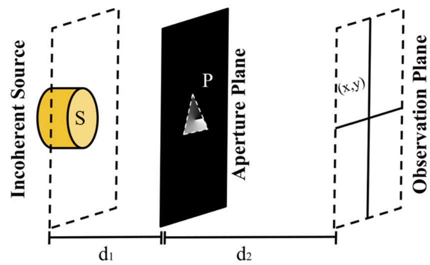


**Fig. 1.** System used for diffraction pattern calculations. $S$ is a spatially incoherent source, the aperture plane contains an equilateral triangular aperture with a phase dependent spatial function of the form $\exp(im\phi)$ and the resulting intensity pattern $I(x, y)$ is displayed in the observation plane.

transmission of information in such media [11–13]. Here, a theoretical extension of the proposed triangular aperture experiment to measure the topological charge is carried out by studying the diffraction of a partially coherent beam through a triangular aperture. In this way, we hope to grasp the limitations of the method as well as to establish coherence range areas that will provide visible enough results that allows one to count the number of intensity maxima. It should be pointed out that we are not considering speckle fields into our system. It is known that second order field correlations with speckles are not suitable regarding the identification of topological charges in diffraction patterns, although they can be useful in intensity correlation measurements [14].

## 2. Theory

When dealing with partial coherent wavefields, the mutual coherence function $\Gamma(\mathbf{r}, \mathbf{r}', \tau) = \langle E(\mathbf{r}, t)E(\mathbf{r}', t+\tau)\rangle$ is used to describe how the fluctuations of the field points $\mathbf{r}$ and $\mathbf{r}'$ are correlated after a time delay $\tau$. In our considerations we suppose a quasimonochromatic stationary light source and neglect temporal effects, as the mean frequency $\bar{\nu}$ of the source is assumed to be much smaller than its bandwidth $\Delta\nu$. Hence, in the following we shall be concerned with the mutual intensity $J(\mathbf{r}, \mathbf{r}') = \Gamma(\mathbf{r}, \mathbf{r}', 0)$ or its normalized version, the complex coherence factor $\mu(\mathbf{r}, \mathbf{r}')$ [15]. Our interest here is in the partially coherent field generated by an extended uniform source, after it passes through a spatial dependent phase mask. Such system was studied initially by Swartzlander and coworkers who have shown that the cross-correlation function $\chi(\mathbf{r}) = J(\mathbf{r}, -\mathbf{r})$ retains the topological charge information in a dislocation ring as opposed to the transmitted intensity which assumes no zero amplitude values [16]. One interesting result was that the radius of the dislocation ring increased as the coherence of the source decreased, implying that one could measure the value of the topological charge of very highly incoherent sources. In [17] it was shown numerically that the number of dislocation rings in the cross-correlation function was equal to the topological charge value and in [18] the authors compared the dislocation rings in the cross-correlation function with that already presented in the uniform incoherent circular source. Here we propose a slightly different setup in which the spatial phase dependence is incorporated into a triangular aperture. It should be remarked that phase singularities of the mutual coherence function were also studied by several authors [19–22].

The physical system is depicted in Fig. 1. The calculation was divided in two parts. In the first part we specify the geometrical properties of the incoherent source $S$, which in our case it was considered to be either a circular source with radius $R$ or a square source with side length $b$. Light from the incoherent source passes through the aperture plane $P$, located at a distance $d_1$ from the incoherent source plane, which is composed of an equilateral triangular aperture with a phase dependent spatial function of the form $\exp(im\phi)$ where $m$ is an integer also know as the topological charge. In the second part of the calculation the field is propagated to the observation plane $(x, y)$ and the diffraction pattern is calculated by solving an integral transform as described next.

In order to find the desired intensity pattern, we must first calculate the complex coherence factor $\mu$ that is created by the incoherent source at the aperture plane. One way to accomplish this task is to use the Van Cittert–Zernike theorem [15]. The theorem states that the complex coherence factor is proportional to the two dimensional Fourier transform of the intensity profile $I(\zeta, \eta)$ of the incoherent source:

$$\mu \propto \int\int I(\zeta,\eta)\exp\left[i\frac{2\pi}{\lambda_m d_1}(x'\zeta + y'\eta)\right]d\zeta d\eta, \tag{1}$$

where $(\zeta, \eta)$ is the plane containing the source, $(x', y')$ the aperture plane, $\lambda_m$ the mean wavelength and $d_1$ the distance indicated in Fig. 1. It was assumed that the distance $d_1$ is much larger than the distance between any pair of points belonging to the source (the actual physical source, not the source plane, of course). With the complex coherence factor in hands, it is necessary to calculate the autocorrelation function of the transmittance aperture, given by

$$\wp(x', y') = \int\int t\left(u - \frac{x'}{2}, v - \frac{y'}{2}\right) t^*\left(u + \frac{x'}{2}, v + \frac{y'}{2}\right) du dv, \tag{2}$$

where $t(x', y')$ is equal to $\exp(im\phi')$ if $(x', y')$ lies inside the triangle or it vanishes otherwise, as indicated in Fig. 2. Now that both, the complex coherence factor and the aperture autocorrelation function, are known we are able to obtain the intensity pattern at the observation plane $(x, y)$:

$$I(x, y) \propto \int\int \wp(x', y')\mu(x', y')\exp\left[i\frac{2\pi}{\lambda_m d_2}(xx' + yy')\right]dx'dy', \tag{3}$$

which is expressed as a two dimensional Fourier transform of the product of the functions $\wp$ and $\mu$. This result is also known as Schell's theorem. It was assumed again that the distance $d_2$ is much larger than the mean wavelength and the characteristic size of the aperture.

## 3. Results and discussion

Consider first a circular incoherent source of radius $R$ described by the intensity profile $I(\zeta, \eta) = 1$ if $(\zeta^2 + \eta^2)^{1/2} \le R$ and $I(\zeta, \eta) = 0$ if $(\zeta^2 + \eta^2)^{1/2} > R$. To calculate the complex coherence factor we use Equation (1), resulting in

$$\mu^{circ}(x', y') \propto \frac{J_1\left(\frac{2\pi R}{\lambda_m d_1}\sqrt{x'^2 + y'^2}\right)}{\frac{2\pi R}{\lambda_m d_1}\sqrt{x'^2 + y'^2}}, \tag{4}$$

where $d_1$ is the distance between the source and the aperture planes (Fig. 1). It is useful to define the coherence area $A_{coh}$ by the following expression

$$A_{coh} = \int\int_{-\infty}^{\infty} |\mu(x', y')|^2 dx'dy'. \tag{5}$$

For a circular incoherent source of radius $R$ we have $A^{circ}_{coh} = (\lambda_m d_1)^2/\pi R^2$. A naive interpretation of the coherence area is that higher values of $A_{coh}$ describes higher coherent fields. By introducing the aperture area, $A_{ap} = \sqrt{3}a^2/2$, where $a$ is the length side of the triangle, we can define the coherence ratio $J$ as

$$J = \frac{A_{coh}}{A_{ap}}, \tag{6}$$



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