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# Vibration driven random walk in a Chladni experiment

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ABSTRACT

Drifting of sand particles bouncing on a vibrating membrane of a Chladni experiment is characterized statistically. Records of trajectories reveal that bounces are circularly distributed and random. The mean length of their horizontal displacement is approximately proportional to the vibration amplitude above the critical level and amounts about one fourth of the corresponding bounce height. For the description of horizontal drifting of particles a model of vibration driven random walk is proposed that yields a good agreement between experimental and numerically simulated data.



## 1. Introduction

Accumulation of sand particles at nodal lines of vibrating surfaces was first observed by Robert Hook in 1680 and utilized a century later by Ernst Chladni in a series of inventions for visualization of vibrations that have significantly contributed to the development of vibration science and acoustics [1,2]. Since variations of Chladni's technique are still commonly used in the production of acoustic instruments [2], it is rather surprising that so far the accumulation phenomenon has remained physically rather incompletely explained. This fact appears even more remarkable in regard to the extent of theoretical and experimental explorations of bouncing phenomena in the development of chaotic dynamics [4–7], and more recent explorations of bouncing and granular flow phenomena [8–18]. The reason for this incompleteness is the complexity of the governing dynamics whose modeling requires a consideration of the chaotic phenomena involved, stochastic properties of particle shape and vibrating surface roughness, dissipation of energy by friction or collisions between particles, and substrate wave motion. This variety prevents a proper analytical description of the Chladni pattern formation by a single dynamic model.

Merely by intuition one can presume that, irrespectively of the mechanism details [2,3], the particles drift by successive bounces from regions of intense vibrations to regions around nodal lines, where the vibration amplitude vanishes. The aim of this letter is to provide an experimental support for such reasoning based upon the measured statistical characteristics of the phenomenon. For this purpose the movement of sand particles in a simple Chladni experiment on a circular elastic membrane is explored. The goal is to provide a simple statistical basis by which the formation of Chladni patterns could be modeled by methods developed for the random walk phenomena [19]. In order to avoid treatment of interaction between particles, we consider just examples with low surface density of particles and characterize the properties of bouncing by experiments with individual particles.

Recent investigations of bouncing phenomena have shown [8–18] that their mechanism is rather complex [4–6] and could hardly be applied in a simple modeling of pattern formation. Chladni experiments are usually performed with sand particles of rather irregular form which introduces randomness. Therefore, the evolution of a Chladni pattern is a stochastic process and we first describe its properties statistically based upon experimental data. The bouncing includes vertical and horizontal displacements. The latter lead to the formation of the Chladni pattern and we utilize the horizontal component of particle displacement as the basis for our description.

The dynamics of bouncing involves tossing of particles by the vibrating surface with acceleration $a(\vec{r}) = \omega^2 z(\vec{r})$, where $\omega$ is the angular frequency and $z(\vec{r})$ the vibration amplitude at the position $\vec{r}$, as well as falling with acceleration of gravity $g$. The bouncing commences above the critical level $z_c = g/\omega^2$ where $a(\vec{r})$ exceeds $g$ and therefore, we describe the forced bouncing in terms of the relative acceleration amplitude $A(\vec{r}) = (a(\vec{r}) - g)/g$, that equals the relative vibration amplitude above the critical level $A(\vec{r}) = z(\vec{r})/z_c - 1$. The bouncing is present if $A > 0$, and absent if $A \leq 0$. At nodal lines $A$ is close to zero and the bouncing excited in regions with $A > 0$ terminates there.

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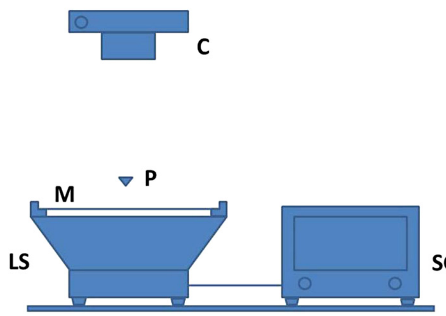


**Fig. 1.** Scheme of the experimental system. SG – signal generator, LS – loudspeaker, C – camera, M – membrane, P – particle.

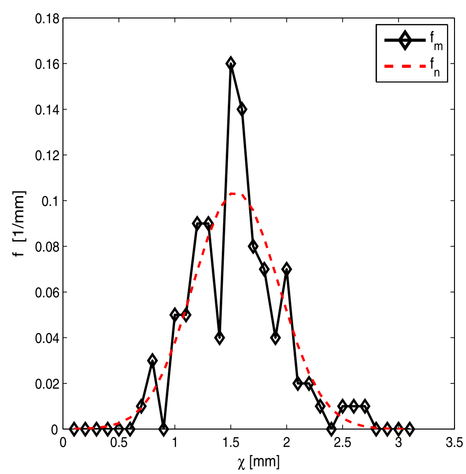


**Fig. 2.** Probability density $f = dN/(Nd\chi)$ of particle height $\chi$. Solid line: $f_m$ – estimated from measured data, dashed line: $f_n$ – corresponding normal distribution.

## 2. Experiment and analysis

The experiment is performed on the experimental system shown in Fig. 1. Bouncing of particles takes place on a circular rubber membrane of radius $r_0 = 152$ mm, thickness 1 mm, density 1.134 kg/dm$^3$, and surface tension 0.194 N/mm. It is tightened into a frame that is mounted horizontally above the loudspeaker. The resonant vibration of the membrane first fundamental mode is excited by the air pressure from a loudspeaker driven by a sinusoidal voltage of frequency $f = 36.8$ Hz provided by a signal generator. With respect to the properties of resonance phenomena, we assume that in this case the radial dependence of the vibration amplitude is given by $z(r) = z_0 J_0(2.4r/r_0)$, where $z_0$ is the amplitude in the center, $J_0$ the Bessel function, and 2.4 its first zero. The amplitude of the excitation signal is set so that the critical vibration amplitude $z_c$ of 0.18 mm is reached at the critical radius $r_c$ of 95 mm. In this case $z_0$ equals $2z_c$ and the span of $A$ extends from 0 to 1.

The testing ensemble includes $N = 30$ quartz particles of approximately tetrahedral form. The probability density function (PDF) $f(\chi) = dN/(Nd\chi)$ of their height $\chi$ is shown in Fig. 2. It is approximately normal with the mean value $\langle\chi\rangle = 1.55$ mm and standard deviation $\Delta\chi = 0.38$ mm.

In the experiments the bouncing of single particles is followed. A particle is put to the center of the membrane at rest and then the vibration is excited by switching on the signal generator. The trajectories of particles are recorded by a photo camera at time intervals $\delta t = 1/3$ s, and the entire record is comprised of 45 images taken within 15 s. From the ensemble of the recorded horizontal position vectors $\vec{r}_n(\tau)$, with $1 \le n \le 30$ and $1 \le \tau \le 45$, we first determine the ensemble of corresponding relative radius $R_n(\tau) = r_n(\tau)/r_c$ and amplitudes $A_n(\tau)$. In addition, the successive displacement samples $\vec{s}_n(\tau) = \vec{r}_n(\tau + 1) - \vec{r}_n(\tau)$ in the exposure times $t = (\tau - 1)\delta t$ are also determined. The corresponding normalized displacement vectors are $\vec{S}_n(\tau) = \vec{s}_n(\tau)/r_c$. The mean over the ensemble of particles $< \ldots > = \sum_n(\ldots)/N$ and the standard deviation $\Delta(\ldots) = \sqrt{\text{var}(\ldots)}$ of variables $R$, $A$, and $S$ are further considered as basic characteristics of the bouncing phenomenon.

Fig. 3a) shows four sample trajectories from the center of membrane toward the critical radius, whereas Fig. 3b) shows the time dependence of the corresponding relative radius $R = r/r_c$. Both figures indicate stochastic character of bouncing. Fig. 4a) shows the distribution of the normalized displacement vectors $\vec{S}_n(\tau)$, while Fig. 4b) shows the time dependence of their $x$-components $S_x$ for the sample trajectories shown in Fig. 3a). Some basic properties of $S_x$ are demonstrated in Fig. 5. Fig. 5a) indicates that the values of $S_x$ are approximately symmetrically distributed around zero at a given vibration amplitude $A$, while the spread of the distribution increases with $A$. The relation between the standard deviation $\Delta S_x$ and the mean amplitude $\langle A\rangle$ over the ensemble of particles is presented in Fig. 5b). The corresponding linear regression line $\Delta S_x = \kappa\,\langle A\rangle$, with $\kappa \approx 0.16$, indicates that on average the distribution spread increases linearly with $\langle A\rangle$.

The probability density function (PDF) determined by Parzen's kernel estimator [20] from the data $\vec{S}_n(\tau)$ presented in Fig. 4a)

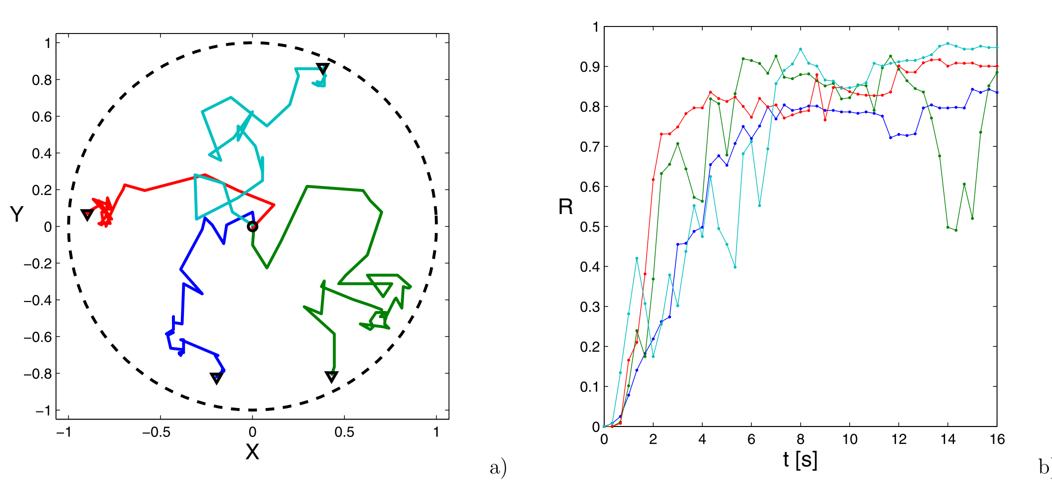


**Fig. 3.** a) Four samples of recorded particle trajectories; dashed circle has radius $r_c$. b) Time dependence of the relative radius $R = r/r_c$ from trajectories in Fig. 2a).