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Energy constraints in pulsed phase control of chaos

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ABSTRACT

Phase control of chaos is a powerful technique but little is known about its physical constraints, relevant for real systems. As a fact, it has not been explored whether this technique can also be applied when the controlling perturbation is not harmonic. Here we apply phase control on a driven double well Duffing oscillator using periodic rectangular pulsed perturbations instead of the classical sinusoidal perturbations. Experimental measurements and numerical simulations show that this kind of perturbation is also able to stabilize the chaotic orbits for an adequate selection of the phase. Furthermore, as the duty cycle of the perturbation (that is, the fraction of the time that the periodically pulsed control is active) is increased, two separate regimes occur. In the first one, the perturbations leading to stabilization of periodic solutions are of constant energy (taken as the product of the duty cycle and the amplitude) and in the second one, a saturation phenomenon occurs, implying that increasing energy values of the perturbations are wasted. Our results unveil the versatility of the pulsed phase control scheme and the importance of energy constraints.

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1. Introduction

The Duffing oscillator is a paradigmatic system in nonlinear dynamics [1–5] and in the context of chaos control [6–8]. Focusing on controlling chaos by means of small periodic perturbations [9–15], it emerges the crucial role of the phase difference between the driving signal necessary to develop chaos and the sinusoidal perturbation necessary for its control. The technique is also known as phase control of chaos and its validity has been tested over different systems [16–18]. Recently phase control of chaos has been revisited from the experimental and theoretical point of view in the single- and double well Duffing Oscillators comparing the different efficiencies when a perturbation is applied to the two terms constituting the quartic potential or to the driving term [19]. Although these investigations have established the amplitude and phase values necessary to achieve stability, still other aspects remain to be explored. For example, it is not clear whether a perturbation with a different functional form, such as a periodic pulsed perturbation made of rectangular-shaped pulses, could lead to the stabilization of the chaotic orbits.

For this reason, here we explore phase control by using periodic rectangular pulsed perturbations. Furthermore, this study provides a simple framework to stress the role of the energy content of the controlling perturbation, given by the product of its strength times its duration (or in other terms the area of the applied control signal) a parameter of paramount importance when considering applications. Notice that our approach to stabilize periodic orbits differs from the Occasional Proportional Feedback (OPF) technique proposed by Hunt [20] and the so-called impulsive control methods [21–24]. OPF relies on an aperiodic pulsed control signal constructed on the chaotic output signal and it can be considered as a one-dimensional version of the OGY method [6]. The impulsive method proposed by Osipov et al. [21] is based on suppressing chaos in the return map associated with the trajectories of a continuous system. Impulsive control of chaos is associated with impulsive differential equations describing evolution processes where the state variables are subjected to jumps at some discrete times. This kind of control is particularly attractive for modulating digital information on a chaotic carrier signal for secure communications [22–24].

Thus, in our pulsed phase control scheme, in addition to the phase relationships with respect to the driving term, we focus the attention on the energy content of the applied perturbation related on both its strength and time duration, allowing us to explore in a systematic way the relation between these two quantities. Impor-

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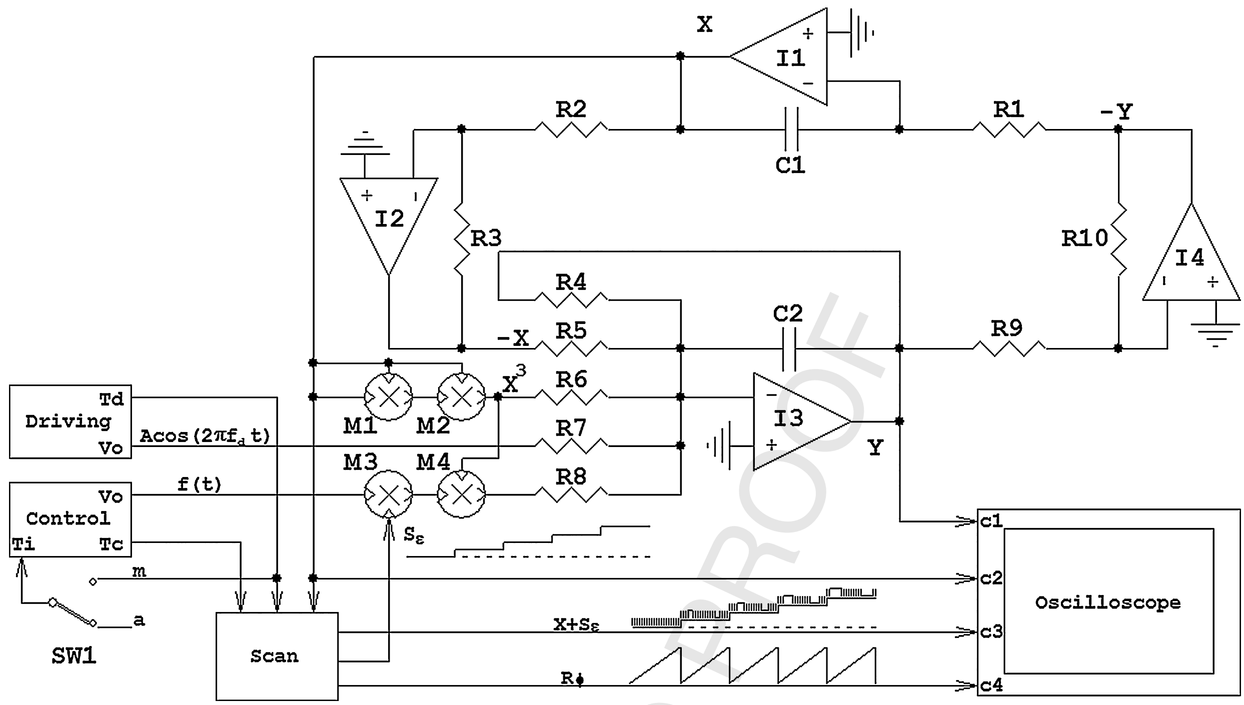

Fig. 1. Schematic set-up of a double well Duffing oscillator with phase and perturbation strength sweeping. It includes the Duffing oscillator with outputs x and y and electronic components specified as follow: I1 and I3 integrators (LT1114); I2 and I4 inverting amplifiers (LT1114); C1 = C2 = 10 nF; R1 = R2 = R3 = R5 = R7 = R8 = R9 = R10 = 10 k; R4 = 40 k; R6 = 1, 6 k; M1, M2, M3 and M4 multipliers (MLT04). SW1 is a 2 poles one track switch allowing the selection of automatic (a) and manual (m) phase and amplitude sweeping. Driving block generates the sinusoidal driving signal. Control block generates a rectangular pulse. Scan block allows automatic scanning of the phase ϕ and the perturbation strength ε through the ramp signal R_ϕ and the staircase signal S_ε. The trigger output T_d of the Driving generator triggers the Control generator via the trigger input Ti as the manual sweeping operation is selected by SW1.

tantly, minimal energy perturbations are crucial for a number of practical applications, such as in biomedical applications of chaos control [25].

2. Experimental validation

Pulsed phase control of chaos is first tested in an analog implementation of the Duffing Oscillator shown in Fig. 1.

The electronic scheme includes a double well Duffing oscillator as already described in Ref. [19] and three blocks, Driving, Control and Scan. The first one provides a sinusoidal signal to maintain the oscillator in a chaotic regime. The second one provides a suitable rectangular pulse with an adjustable phase shift with respect to the driving signal and applied to the cubic nonlinearity of the oscillator. The last block (Scan), triggered by the first two, generates two signals, that is, a linear ramp R_ϕ for a phase variation of 2π and a 32 level staircase signal S_ε (constant in amplitude during one phase sweep) allowing us to perform a sweeping of the perturbation strength ε. The x and y signals from the Duffing oscillator together with the phase-ramp and $x + S_\varepsilon$ signal are monitored on a four trace oscilloscope.

In our experiment, phase control has been applied on the cubic term of the Duffing oscillator according to the following equations:

$$\dot{x} = y \tag{1}$$

$$\dot{y} = -\gamma y + x + \big(1 + f(t)\big)x^3 + A\cos(2\pi f_d t) \tag{2}$$

where $\gamma = 0.25$ is the damping constant, $A = 0.41$ is the amplitude of the sinusoidal driving signal with frequency f_d and the pulsed control perturbation is a square wave of period $T = 1/f_c$ and amplitude ε, so that $f(t) = \varepsilon$ for $t = [-b/2, b/2]$ and 0 during the rest of the period as shown in Fig. 2. The parameter b is related to the duty cycle D of the square wave through the relation $D = b/T$. A relative phase ϕ of the pulsed perturbation $f(t + \phi)$

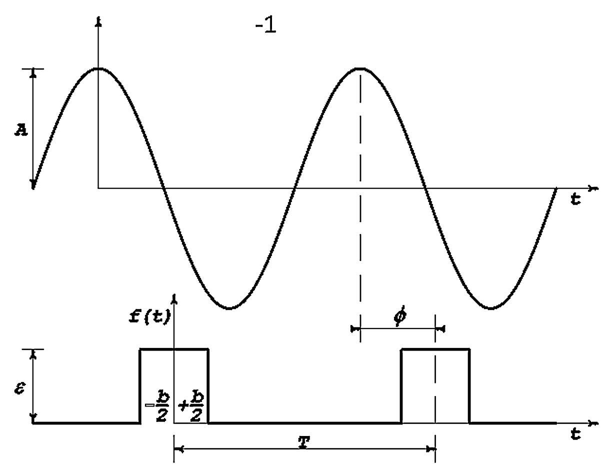

Fig. 2. Periodic perturbation $f(t)$ of period $T = 1/f_c$ representing a rectangular pulse of width b and height ε. The duty cycle is defined as $D = b/T$. The phase difference is defined by considering the maximum of the sinusoidal signal and the midpoint of the pulsed perturbation.

with respect to the driving can be achieved by selecting the frequency of the control signal as $f_c = f_d + 1/T_{sw}$ where T_{sw} is the sweeping phase period during which a phase variation of 2π occurs. In the experiment $f_d = 1592.500$ Hz and $T_{sw} = 2$ s.

Using the sweeping technique for the phase ϕ and relative perturbation strength ε described above it is possible to obtain the stability domains directly on a digital scope (Tektronix TDS7104). In Fig. 3 three stability diagrams are reported for different values of the duty cycle together with the x–y representation of the two stabilized attractors (two period two solutions around the fixed points at $x = \pm 1$) and a not stabilized chaotic attractor. Hence, we can conclude that this type of periodic pulsed perturbation can also produce regimes of periodic dynamics. In these representa-

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