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Extraordinary lateral beaming of sound from a square-lattice phononic crystal

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ABSTRACT

This work revisits the sound transmission through a finite phononic crystal of square lattice. In addition to a direct, ordinary transmission through the sample, an extraordinary lateral beaming effect is also observed. The phenomenon stems from the equivalence of the states located around the four corners of the first Brillouin zone. The experimental result agrees well with the theoretical prediction. The lateral beaming behavior enables a simple design for realizing acoustic beam splitters.

In recent decades, the sound propagation in artificially structured composites, e.g., acoustic metamaterials [1–16] and phononic crystals (PCs) [17–33], has attracted great interest, since such composites carry many exotic properties unattainable in the conventional homogeneous media. The acoustic metamaterials, made of deep subwavelength units, are famed for their abnormal effective parameters, from the negative mass density and bulk modulus [1–6] to the strong anisotropy [7–10]. Many interesting acoustic devices are proposed based on such unusual properties, e.g., various acoustic lenses [7,8,11,12] and sound cloaks [13–16]. For the PCs, the structural units could be distinguished by the operation wavelength. The strong multiple scattering of sound inside the PC can generate a remarkable modification to the dispersion curve for the propagating mode, in addition to create frequency gaps that forbid sound propagation. Based on the band gap effect and the modified band structure, numerous works have been devoted to realize sound shielding [17–19], waveguiding [20–22], negative refraction [23–28], and directional radiation [29–32].

It is well-known that, the excitation of the Bloch states in a PC can be predicted by an equifrequency contour (EFC) analysis based on the principle of the quasi-momentum conservation along the PC boundary [33]. Thus, for a flat PC slab of infinite length, a sound beam will transmit through the sample at an angle parallel to the incident one, as long as the frequency is not high enough to produce additional diffraction branches. This sound transmission (directly through the sample) can be called 'ordinary transmission'. If the PC slab has a finite width, one may wonder whether the Bloch states can be coupled out from its lateral sides as well. This problem, despite fundamental, has not been well explored so far. In this work, we study the transmission of a Gaussian beam through a finite PC sample of square lattice, of which the surface normal is selected along the ΓM direction. In addition to the ordinary transmission beam, we have also observed an extraordinary lateral beaming (ELB) behavior. The phenomenon is explained by the equivalence of the states located around the four corners of the first Brillouin zone. Throughout this paper, all simulations are carried out by the commercial finite-element software (COMSOL MULTIPHYSICS). The numerical results are confirmed by our underwater ultrasonic experiments. Potential applications for this acoustic ELB effect can be anticipated, such as to design sound splitters [34–37].

The PC under consideration consists of a square array of steel cylinders immersed in water, where the lattice constant $a = 1.4$ mm and the diameter of the cylinder $d = 1.0$ mm. The material parameters used are: the mass density $\rho = 7760$ kg/m^3, the longitudinal velocity $v_l = 6010$ m/s, and the transverse velocity $v_t = 3230$ m/s for steel; and the mass density $\rho_0 = 1000$ kg/m^3 and the sound speed $c_0 = 1490$ m/s for water. In Fig. 1(a) we present the band structure of the PC along the ΓX and ΓM directions, which shows a band-edge frequency of 0.592 MHz at the corner points of the first Brillouin zone. In Fig. 1(b) we illustrate the physics mechanism behind the ELB effect through the frequently-used EFC analysis. The circles show the EFCs for several frequencies slightly below the band-edge. We consider a finite PC sample with its surface normal along the ΓM direction. For a Gaussian beam incident normally onto the sample from the bottom, it is natural that the Bloch states S_1 and S_4 are stimulated according

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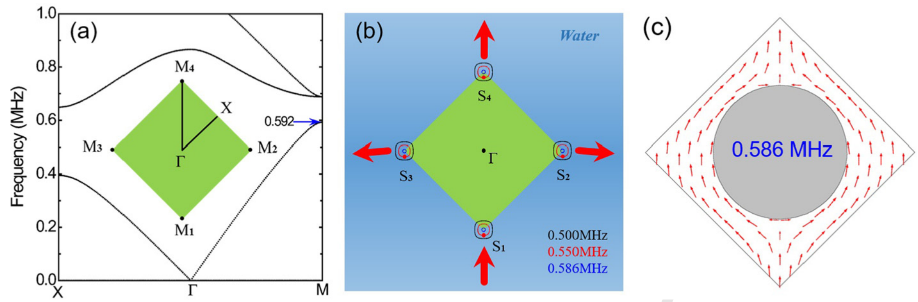

Fig. 1. (a) Directional band structure for a PC arranged by a square lattice array of steel cylinders in water. Inset: the first Brillouin zone of the square lattice, with M_1–M_4 labeling four equivalent corner points. (b) Schematic illustration of the acoustic ELB effect based on the EFC analysis. A Gaussian beam incident vertically will excite four equivalent Bloch modes S_1–S_4, and then couple out from the front and bilateral sides of the finite PC slab. The bold arrows indicate the directions of the incident and outgoing beams. (c) Energy flow (red arrows) distribution of the eigenfield in a unit cell, evaluated for the frequency 0.586 MHz. The gray region indicates the solid cylinder. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

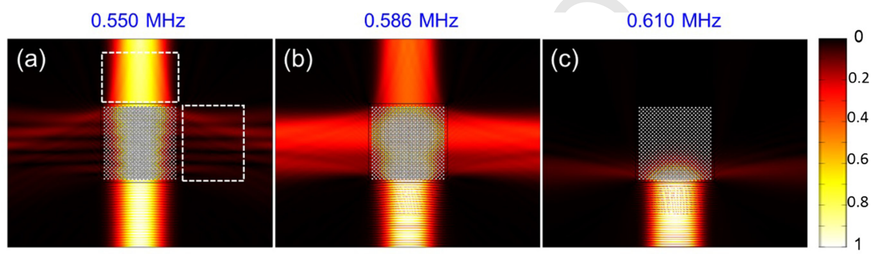

Fig. 2. The pressure amplitude distributions for the frequencies (a) 0.550 MHz, (b) 0.586 MHz and (c) 0.610 MHz, respectively, simulated by a spatial Gaussian beam normally from the bottom. The white frames in (a) indicate the spatial regions to be scanned experimentally.

to the quasi-momentum conservation along the horizontal boundary between the PC and water. A beam will be coupled out from the top boundary of the PC, leading to the ordinary transmission behavior. In addition, the Bloch states S_2 and S_3 will be excited as well, since they are equivalent to the states S_1 and S_4 (connected by reciprocal lattice vectors). Therefore, at the lateral boundaries the acoustic waves can be coupled out along a nearly horizontal direction. One may wonder how the vertical energy flow inside the PC is coupled into the lateral beams along the horizontal direction. Microscopically, this can be understood by the energy flow distribution displayed in Fig. 1(c), evaluated for the eigen-field at 0.586 MHz in a single unit cell. Although the energy flow points to the vertical direction as a whole, it shows clearly a horizontal component, which contributes to the ELB phenomenon for a sample with finite width.

To confirm the above theoretical prediction, we consider a PC sample of square shape (made of 761 cylinders in total), as shown in Fig. 2. The surface normal is selected along the ΓM direction. A Gaussian beam is launched vertically onto the sample from the bottom. Three typical frequencies are considered. The pressure pattern in Fig. 2(a) displays that, for the frequency 0.550 MHz, there is indeed a portion of sound energy leaked out from the left and right sides of the sample, despite the fact that most of the sound energy transmits through the front facet directly. The bilateral emission gets stronger as the band-edge frequency is approached. For example, at the frequency 0.586 MHz [Fig. 2(b)], the field amplitudes become comparable for the three outgoing beams. As the frequency increases further, e.g., at 0.610 MHz, all three beams weaken rapidly and the acoustic wave is almost totally reflected from the input end owing to the band gap effect. (The narrow beams appearing in the lateral sides stem from the weak penetration of sound into the sample.) By integrating the pressure amplitude for each output facet, we have evaluated the transmission coefficient normalized by that of incident beam. Fig. 3 shows the transmission spectra, where the black and red lines correspond to the amplitude transmissions through the front and right sides, respectively. As predicted, the spectrum for the lateral transmis-

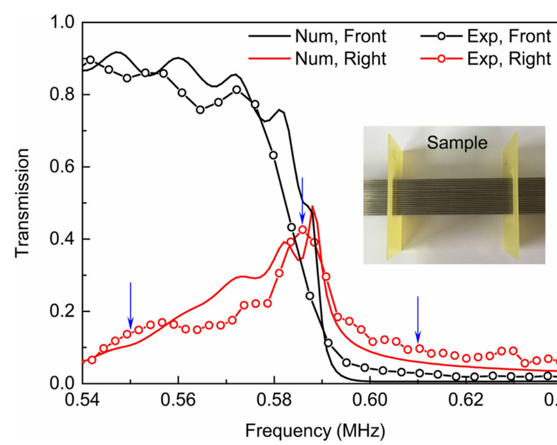

Fig. 3. Numerical and experimental transmission spectra for the outgoing beams from the front and right sides, respectively. The inset shows a photograph of the experimental sample, where the steel cylinders (gray) are precisely arranged by penetrating them through two plastic plates (yellow) with square-lattice holes. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

sion shows a pronounced peak (as high as ∼0.49) near the band-edge frequency. Interestingly, the bilateral transmission decreases rapidly as the frequency deviates from the band-edge. The narrow band effect can be qualitatively explained as follows. The (moving-upward) Bloch state can be decomposed into a series of Fourier components (connecting with reciprocal lattice vectors). Assume that the index i ($i = 1, 2, 3, 4$) stands for the Fourier component with wavevector close to one of the four corner points M_i [as labeled in Fig. 1(a)]. Near the band-edge frequency, the weights of the four Fourier components are comparable because of the strong Bragg scattering, which leads to strong radiation along the horizontal direction. As the frequency decreases, the bilateral beaming effect weakens greatly since the weights of the Fourier components $i = 2$ and $i = 3$ reduce rapidly.

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