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Influence of field and geometric configurations on the mode conversion characteristics of hybrid waves in a magnetoplasma slab

Myoung-Jae Lee [a,b], Young-Dae Jung [c,d,*]

[a] Department of Physics, Hanyang University, Seoul 04763, South Korea
[b] Research Institute for Natural Sciences, Hanyang University, Seoul 04763, South Korea
[c] Department of Applied Physics and Department of Bionanotechnology, Hanyang University, Ansan, Kyunggi-Do 15588, South Korea
[d] Department of Electrical and Computer Engineering, MC 0407, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0407, USA

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ABSTRACT

We explore the mode conversion characteristics of electrostatic hybrid surface waves due to the magnetic field orientation in a magnetoplasma slab. We obtain the dispersion relations for the symmetric and anti-symmetric modes of hybrid surface waves for two different magnetic field configurations: parallel and perpendicular. For the parallel magnetic field configuration, we have found that the symmetric mode propagates as upper- and lower-hybrid waves. However, the hybrid characteristics disappear and two non-hybrid waves are produced for the anti-symmetric mode. For the perpendicular magnetic field configuration, however, the anti-symmetric mode propagates as the upper- and lower-hybrid waves and the symmetric mode produces two non-hybrid branches of waves.

The investigation of surface waves in a plasma has drawn much interest since the dispersion relation since it provides useful information on various plasmas spatially separated from their surroundings such as a vacuum or dielectrics [1–10]. In recent years, there has been a great deal of interest in waves in bounded dusty plasmas [11–14] as well as in bulk plasmas [15–18] since dust grains are often found in space and laboratory. The existence of electrostatic hybrid oscillations was previously reported for the surface wave propagation in the semi-bounded (semi-infinity) magnetized dusty plasma [19]. In Ref. [19], the frequencies of the electrostatic upper- and lower-hybrid resonance oscillations in a semi-bounded plasma are found to be independent of the orientation of the external magnetic field but those frequencies can be enhanced by the increase of the magnetic field strength. However, the propagation of surface waves in the plasma slab would be quite different from the case of semi-bounded plasma. Since a plasma slab can support propagation for the symmetric and anti-symmetric modes depending on wave conditions on the two boundary surfaces of the slab [2,20], the dispersion relations for both modes can be derived for each boundary condition. In this paper, we are motivated to investigate the surface waves propagating in a magneto dusty plasma including the effects of slab thickness as well as the magnetic field strength and orientation. The dependence on the field orientation of the waves for the symmetric and anti-symmetric modes in a dusty slab plasma has not been known yet. In this work, we choose the frequency range above the ion cyclotron frequency but less than the electron cyclotron frequency.

We consider a dusty plasma slab with the sharp boundaries at $x = 0$ and $x = L$ such that the characteristic length of plasma is much greater than the scale length of the inhomogeneity. Then, the specular reflection condition in which the particles undergo a mirror reflection such that $f_1(v_x, v_y, v_z, t)|_{x=0} = f_1(-v_x, v_y, v_z, t)|_{x=0}$ and $f_1(v_x, v_y, v_z, t)|_{x=L} = f_1(-v_x, v_y, v_z, t)|_{x=L}$, where f_1 is the perturbed plasma distribution function, can be used as the boundary condition for the study of surface waves [1,2]. This boundary condition yields the dispersion equation for electrostatic surface waves propagating in the z direction in an isotropic plasma slab represented by [21]

$$1 + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dk_x k_z}{k^2 \varepsilon_l(\omega, k)} \left(\frac{1 \mp e^{ik_x L}}{1 \pm e^{ik_x L}} \right) = 0, \qquad (1)$$

where ω is the wave frequency, k_x and k_z are the x- and z-components of the wave vector $\mathbf{k}$, respectively, ε_ℓ is the longitudinal component of the plasma dielectric permittivity, and L is the slab thickness, and $k(= |\mathbf{k}|) = \sqrt{k_x^2 + k_z^2}$. Since the y-component

* Corresponding author at: Department of Applied Physics and Department of Bionanotechnology, Hanyang University, Ansan, Kyunggi-Do 15588, South Korea. Fax: +82 31 400 5457.
E-mail address: ydjung@hanyang.ac.kr (Y.-D. Jung).

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of wave vector has a translational invariance, it is ignored in our geometry without loss of generality. The double signs in Eq. (1) denote the two modes of surface waves in a plasma slab: symmetric (upper sign) and anti-symmetric (lower sign) modes. For the symmetric mode, we have $E_z(x=L)=E_z(x=0)$, whereas for the anti-symmetric mode $E_z(x=L)=-E_z(x=0)$.

When the parallel magnetic field $\mathbf{B}_0=B_0\hat{\mathbf{z}}$ is applied to the boundary surfaces, the longitudinal plasma dielectric permittivity in dusty plasma for $kv_{T\alpha}, \omega_{cd}, \omega_{ci} \ll \omega \ll \omega_{ce}$ is obtained as follows [22]:

$$\varepsilon_{l,\parallel}(\omega, k_x, k_z) = 1 + \frac{\omega_{pe}^2 k_x^2}{\omega_{ce}^2 k^2} - \frac{\omega_{pe}^2 k_z^2}{\omega^2 k^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}, \tag{2}$$

where $\omega_{p\alpha} = (4\pi n_\alpha q_\alpha/m_\alpha)^{1/2}$ is the plasma frequency of species α ($=e,i,d$ for electron, ion and dusty grain, respectively) and $\omega_{c\alpha}=q_\alpha B_0/m_\alpha c$ is the cyclotron frequency of species α. Plugging Eq. (2) into Eq. (1), the dispersion equation becomes

$$1 + \frac{k_z}{\pi\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)} \int_{-\infty}^{\infty} \frac{dk_x}{k_x^2 + F_\parallel^2(\omega) k_z^2} \left(\frac{1 \mp e^{ik_x L}}{1 \pm e^{ik_x L}}\right) = 0, \tag{3}$$

where the symbol $F_\parallel(\omega)$ in the denominator is defined as

$$F_\parallel^2(\omega) = \frac{1 - \frac{\omega_{pe}^2+\omega_{pi}^2+\omega_{pd}^2}{\omega^2}}{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}}. \tag{4}$$

Since the integral in Eq. (3) vanishes as $k_x \to \infty$, we calculate the integral by applying the Cauchy residue theorem with integration being performed along a closed contour. By picking up the pole at $k_x = F_\parallel(\omega) k_z$, we find the dispersion relation for the symmetric and the anti-symmetric modes of surface waves in the magnetoplasma slab such that

$$\left[\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\left(1 - \frac{\omega_{pe}^2+\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\right]^{\frac{1}{2}} + \tanh\left[\frac{1}{2}F_\parallel(\omega) k_z L\right] = 0 \quad \text{(symmetric mode)} \tag{5}$$

and

$$\left[\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\left(1 - \frac{\omega_{pe}^2+\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\right]^{\frac{1}{2}} + \coth\left[\frac{1}{2}F_\parallel(\omega) k_z L\right] = 0 \quad \text{(anti-symmetric mode).} \tag{6}$$

For long-wavelength oscillations, when $F_\parallel(\omega) k_z L \ll 1$, the arguments in the hyperbolic functions can be expanded and terms with higher order than the primary can be truncated to obtain the wave frequency. We see that there exist two kinds of hybrid oscillations for the symmetric mode. We denote the upper-hybrid wave as $\bar{\omega}_{\parallel S}^{UH}$ and the lower-hybrid wave as $\bar{\omega}_{\parallel S}^{LH}$ where $\bar{\omega} = \omega/\omega_{pi}$ is the scaled wave frequency and $\bar{\omega}_{\parallel S}^{UH}$ and $\bar{\omega}_{\parallel S}^{LH}$ are obtained by

$$\bar{\omega}_{\parallel S}^{UH} = \left(\frac{1+\frac{\omega_{pd}^2}{\omega_{pi}^2}}{1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{1}{2}\bar{k}_z \bar{L}}\right)^{\frac{1}{2}} \quad \text{(upper-hybrid wave)} \tag{7}$$

and

$$\bar{\omega}_{\parallel S}^{LH} = \left(\frac{1+\frac{\omega_{pd}^2}{\omega_{pi}^2}}{1+\frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{1}{2}\bar{k}_z \bar{L}}\right)^{\frac{1}{2}} \quad \text{(lower-hybrid wave).} \tag{8}$$

We also find another two branches of non-hybrid waves for the anti-symmetric mode: $\bar{\omega} = \bar{\omega}_{\parallel AS+}$ and $\bar{\omega} = \bar{\omega}_{\parallel AS-}$ which are obtained in the form,

$$\bar{\omega}_{\parallel AS+} = \left(\frac{1+\frac{\omega_{pe}^2}{\omega_{pi}^2}+\frac{\omega_{pd}^2}{\omega_{pi}^2}}{1+\frac{2}{\bar{k}_z\bar{L}}}\right)^{\frac{1}{2}} \tag{9}$$

and

$$\bar{\omega}_{\parallel AS-} = \left(\frac{1+\frac{\omega_{pe}^2}{\omega_{pi}^2}+\frac{\omega_{pd}^2}{\omega_{pi}^2}}{1-\frac{2}{\bar{k}_z\bar{L}}}\right)^{\frac{1}{2}}, \tag{10}$$

where $\bar{k}_z \equiv k_z\lambda_{De}$ and $\bar{L} \equiv L/\lambda_{De}$ are the scaled wave number and scaled slab thickness. It is interesting to note that the hybrid mode is observed for the symmetric parallel propagation to $\mathbf{B}_0$ but it disappears for the anti-symmetric propagation.

If a perpendicular magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{x}}$ is applied to slab, the longitudinal plasma dielectric permittivity in dusty plasma for $kv_{T\alpha}, \omega_{cd}, \omega_{ci} \ll \omega \ll \omega_{ce}$ is written as

$$\varepsilon_{l,\perp}(\omega, k_x, k_z) = 1 + \frac{\omega_{pe}^2 k_z^2}{\omega_{ce}^2 k^2} - \frac{\omega_{pe}^2 k_x^2}{\omega^2 k^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}. \tag{11}$$

Equation (11) can be inserted into the dispersion equation and the integral can be executed again by residue theorem to yield the following dispersion relations,

$$\left[\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\left(1 - \frac{\omega_{pe}^2+\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\right]^{\frac{1}{2}} + \tanh\left[\frac{1}{2}F_\perp(\omega) k_z L\right] = 0 \quad \text{(symmetric mode)} \tag{12}$$

and

$$\left[\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\left(1 - \frac{\omega_{pe}^2+\omega_{pi}^2+\omega_{pd}^2}{\omega^2}\right)\right]^{\frac{1}{2}} + \coth\left[\frac{1}{2}F_\perp(\omega) k_z L\right] = 0 \quad \text{(anti-symmetric mode),} \tag{13}$$

where

$$F_\perp(\omega) = \left(\frac{1+\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2+\omega_{pd}^2}{\omega^2}}{1-\frac{\omega_{pe}^2+\omega_{pi}^2+\omega_{pd}^2}{\omega^2}}\right)^{\frac{1}{2}}. \tag{14}$$

In the limit of $F_\perp k_z L \ll 1$, the arguments in the hyperbolic functions can be expanded and terms with higher order than the primary can be truncated to obtain the wave frequency. Then, two branches of non-hybrid oscillations for the symmetric mode are found as $\bar{\omega} = \bar{\omega}_{\perp S+}$ and $\bar{\omega} = \bar{\omega}_{\perp S-}$ where

$$\bar{\omega}_{\perp S+} = \left(\frac{1+\frac{\omega_{pe}^2}{\omega_{pi}^2}+\frac{\omega_{pd}^2}{\omega_{pi}^2}}{1+\frac{1}{2}\bar{k}_z\bar{L}}\right)^{\frac{1}{2}} \tag{15}$$

and

$$\bar{\omega}_{\perp S-} = \left(\frac{1+\frac{\omega_{pe}^2}{\omega_{pi}^2}+\frac{\omega_{pd}^2}{\omega_{pi}^2}}{1-\frac{1}{2}\bar{k}_z\bar{L}}\right)^{\frac{1}{2}}. \tag{16}$$