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Nonlinear response of the quantum Hall system to a strong electromagnetic radiation

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ABSTRACT

We study nonlinear response of a quantum Hall system in semiconductor-hetero-structures via third harmonic generation process and nonlinear Faraday effect. We demonstrate that Faraday rotation angle and third harmonic radiation intensity have a characteristic Hall plateaus feature. These nonlinear effects remain robust against the significant broadening of Landau levels. We predict realization of an experiment through the observation of the third harmonic signal and Faraday rotation angle, which are within the experimental feasibility.

Integer quantum Hall effect (QHE) is remarkable phenomenon of two dimensional electron gas (2DEG) systems, in which the longitudinal resistance vanishes while the Hall resistance is quantized into plateaus [1]. The static QHE is the hallmark of dissipationless topological quantum transport [2] and despite its long history there is a continuing enormous amount of interest on this effect along various avenues. With the advent of new materials, such as graphene and topological insulators new regimes of QHE have been revealed [3,4]. While static properties of the integer QHE have been well investigated in the scope of linear response theory, the dynamic and nonlinear responses in the quantum Hall system (QHS) in the high-frequency regime are not fully explored. In Ref. [5] considering the quantum dynamics of QHS exposed to an intense high-frequency electromagnetic wave, it is shown that the wave decreases the scattering-induced broadening of Landau levels. Linear response of the QHS in the high-frequency regime has been theoretically examined in Ref. [6]. As was shown in Ref. [6] the plateau structure in the QHS is retained, up to significant degree of disorder, even in the THz regime, although the heights of the plateaus are no longer quantized. Then this effect has been confirmed experimentally in Ref. [7]. Thus, a problem remains as how QHS responded to a strong and high-frequency electromagnetic wave fields, which is the purpose of the present study. In this case it is of interest to study generation of harmonics [8,9] at the interaction of a strong pump wave with the Landau quantized 2DEG.

In the QHS wave-particle interaction can be characterized by the dimensionless parameter $\chi = eE_0 l_B/(\hbar\omega)$, which represents the work of the wave electric field E_0 on the magnetic length $l_B = \sqrt{c\hbar/(eB)}$ (e is the elementary charge, $\hbar$ is Planck's constant, c is the light speed in vacuum, and B is the magnetic field strength) in units of photon energy $\hbar\omega$. The linear response theory is valid at $\chi << 1$. At $\chi \sim 1$ multiphoton effects become considerable. In this paper we consider just multiphoton interaction regime and look for features in the harmonic spectra of the strong wave driven QHS. As a 2DEG system we consider GaAs/AlGaAs single heterojunction. The time evolution of the considered system is found using a nonperturbative numerical approach, revealing that the generated in the QHS harmonics' radiation intensity has a characteristic Hall plateaus feature. The effect remains robust against a significant broadening of Landau levels and takes place for wide range of intensities and frequencies of a pump wave.

We begin our study with construction of the single-particle Hamiltonian which defines the quantum dynamics of considered QHS. The 2DEG is taken in the xy plane ($z = 0$) and a uniform static magnetic field is applied in the OZ direction. We consider an incoming electromagnetic radiation pulse $E(t - z/c)$ propagating in the OZ direction and linearly polarized along the x axis. The incoming wave is assumed to be quasimonochromatic of carrier frequency ω and slowly varying envelope $E_0(t)$. For the 2DEG as realized in GaAs/AlGaAs we have uniform time-dependent electric field $E(t) = E_0(t)\sin\omega t$ and the single-particle Hamiltonian of QHS reads:

$$\mathcal{H}_s = \hbar\omega_B\left(\widehat{a}^{\dagger}\widehat{a} + \frac{1}{2}\right) + \left[\frac{el_B E(t)}{\sqrt{2}}\left(\widehat{b} + i\widehat{a}\right) + \text{h.c.}\right]. \quad (1)$$

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Here $\omega_B = eB/(m^*c)$ is the cyclotron frequency, $m^* = 0.068m_e$ is the effective mass (m_e – the bare electron mass). For the interaction Hamiltonian we use a length gauge describing the interaction by the potential energy. The ladder operators $\widehat{a}$ and $\widehat{a}^\dagger$ describe quantum cyclotron motion, while $\widehat{b}$ and $\widehat{b}^\dagger$ correspond to guiding center motion. These ladder operators satisfy the usual bosonic commutation relations $[\widehat{a}, \widehat{a}^\dagger] = 1$ and $[\widehat{b}, \widehat{b}^\dagger] = 1$. The single free particle Hamiltonian, that is the first term in Eq. (1) can be diagonalized analytically. The wave function and energy spectrum are given by:

$$|\psi_{n,m}\rangle = |n, m\rangle, \tag{2}$$

$$\varepsilon_n = \hbar\omega_B \left(n + \frac{1}{2}\right). \tag{3}$$

Here $|n, m\rangle = |n\rangle \otimes |m\rangle$, with $|n\rangle$ and $|m\rangle$ being the harmonic oscillator wave functions. The eigenstates (2) are defined by the quantum numbers $n, m = 0, 1....$ Here n is the LL index. The LLs are degenerate upon second quantum number m with the degeneracy factor $N_B = \mathcal{S}/2\pi l_B^2$ which equals the number of flux quanta threading the 2D surface $\mathcal{S}$ occupied by the 2DEG. The terms $\sim \widehat{a}E(t)$ in the Hamiltonian (1) describe transitions between LLs, while the terms $\sim \widehat{b}E(t)$ describe transitions within the same LL. These transitions can be excluded from the consideration by the appropriate dressed states for the construction of the carrier quantum field operators. Expanding the fermionic field operator

$$|\widehat{\Psi}\rangle = \sum_{n,m} \widehat{a}_{n,m} |\widetilde{\psi}_{n,m}\rangle \tag{4}$$

over the dressed states

$$|\widetilde{\psi}_{n,m}\rangle = \exp\left[-\frac{i}{\hbar}\frac{el_B}{\sqrt{2}}\int_0^t E(t')dt'\left(\widehat{b}^\dagger + \widehat{b}\right)\right]|\psi_{n,m}\rangle, \tag{5}$$

the Hamiltonian of the system in the second quantization formalism

$$\widehat{H} = \langle\widehat{\Psi}|\,\mathcal{H}_s\,|\widehat{\Psi}\rangle$$

can be presented in the form:

$$\widehat{H} = \sum_{n=0}^{\infty}\sum_{m=0}^{N_B-1} \varepsilon_n \widehat{a}_{n,m}^{+}\widehat{a}_{n,m} + \sum_{n,n'=0}^{\infty}\sum_{m=0}^{N_B-1} E(t)\mathcal{D}_{n,n'}\widehat{a}_{n,m}^{+}\widehat{a}_{n',m}, \tag{6}$$

where $\widehat{a}_{n,m}^\dagger$ and $\widehat{a}_{n,m}$ are, respectively, the creation and annihilation operators for a carrier in a LL state, and $\mathcal{D}_{n,n'}$ is the dipole moment operator:

$$\mathcal{D}_{n,n'} = \frac{iel_B}{\sqrt{2}}\left[\sqrt{n-1}\,\delta_{n-1,n'} + \sqrt{n}\,\delta_{n,n'-1}\right]\frac{\hbar\omega_B}{\varepsilon_{n'} - \varepsilon_n}. \tag{7}$$

Then we will pass to Heisenberg representation where operators obey the evolution equation

$$i\hbar\frac{\partial\widehat{L}}{\partial t} = \left[\widehat{L}, \widehat{H}\right]$$

and expectation values are determined by the initial density matrix $\widehat{D}$: $<\widehat{L}> = Sp\left(\widehat{D}\widehat{L}\right)$. In order to develop microscopic theory of the nonlinear interaction of the QHS with a strong radiation field, we need to solve the Liouville–von Neumann equation for the single-particle density matrix

$$\rho(n_1, m_1; n_2, m_2, t) = <\widehat{a}_{n_2,m_2}^{+}(t)\widehat{a}_{n_1,m_1}(t)> \tag{8}$$

and for the initial state of the quasiparticles we assume an ideal Fermi gas in equilibrium:

$$\rho(n_1, m_1; n_2, m_2, 0) = \frac{\delta_{n_1,n_2}\delta_{m_1,m_2}}{1 + \exp\left(\frac{\varepsilon_{n_1} - \varepsilon_F}{T}\right)}. \tag{9}$$

Including in Eq. (9) quantity ε_F is the Fermi energy, T is the temperature in energy units. As is seen from the interaction term in the Hamiltonian (6) quantum number m is conserved: $\rho(n_1, m_1; n_2, m_2, t) = \rho_{n_1,n_2}(t)\,\delta_{m_1,m_2}$. To include the effect of the LLs broadening we will assume homogeneous broadening of the LLs [10]. The latter can be incorporated into evolution equation for $\rho_{n_1,n_2}(t)$ by the damping term $-i\Gamma_{n_1,n_2}\rho_{n_1,n_2}(t)$ and from Heisenberg equation one can obtain evolution equation for the reduced single-particle density matrix:

$$i\hbar\frac{\partial\rho_{n_1,n_2}(t)}{\partial t} = \left[\varepsilon_{n_1} - \varepsilon_{n_2}\right]\rho_{n_1,n_2}(t) - i\Gamma_{n_1,n_2}\rho_{n_1,n_2}(t)$$
$$- E(t)\sum_n\left[\mathcal{D}_{n,n_2}\rho_{n_1,n}(t) - \mathcal{D}_{n_1,n}\rho_{n,n_2}(t)\right]. \tag{10}$$

For the damping matrix we take $\Gamma_{n_1,n_2} = \Gamma\left(1 - \delta_{n_1,n_2}\right)$, where Γ measures the LL broadening.

Solving Eq. (10) with the initial condition (9) one can reveal nonlinear response of the QHS to a strong radiation pulse. At that one can expect intense radiation of harmonics of the incoming wave-field in the result of the coherent transitions between LLs. The harmonics will be described by the additional generated fields $E_{x,y}^{(g)}$. We assume that the generated fields are considerably smaller than the incoming field $\left|E_{x,y}^{(g)}\right| << |E|$. In this case we do not need to solve self-consistent Maxwell's wave equation with Heisenberg equations. To determine the electromagnetic field of harmonics we can solve Maxwell's wave equation in the propagation direction with the given source term:

$$\frac{\partial^2 E_{x,y}^{(t)}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_{x,y}^{(t)}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial\mathcal{J}_{x,y}(t)}{\partial t}\delta(z). \tag{11}$$

Here $\delta(z)$ is the Dirac delta function and $\mathcal{J}_{x,y}$ is the mean value of the surface current density operator:

$$\widehat{\mathcal{J}}_x(t) = \frac{-2e\hbar}{\sqrt{2}l_B m^*\mathcal{S}}\left\langle\widehat{\Psi}\right|\left(\hat{a}^\dagger + \hat{a}\right)\left|\widehat{\Psi}\right\rangle,$$
$$\widehat{\mathcal{J}}_y(t) = \frac{-2e\hbar}{i\sqrt{2}l_B m^*\mathcal{S}}\left\langle\widehat{\Psi}\right|\left(\hat{a}^\dagger - \hat{a}\right)\left|\widehat{\Psi}\right\rangle. \tag{12}$$

With the help of Eqs. (4) and (8) the expectation value (12) of the total current in components can be written in the following form:

$$\mathcal{J}_x(t) = j_0\sum_{n=0}\sqrt{n+1}\,\mathrm{Re}\rho_{n,n+1}(t),$$
$$\mathcal{J}_y(t) = -j_0\sum_{n=0}\sqrt{n+1}\,\mathrm{Im}\rho_{n,n+1}(t), \tag{13}$$

where $j_0 = -\sqrt{2}e\hbar/\left(\pi l_B^3 m^*\right)$ (here we have taken into account the spin degeneracy factor). The solution to equation (11) reads

$$E_{x,y}^{(t)}(t,z) = E_{x,y}(t - z/c)$$
$$-\frac{2\pi}{c}\left[\theta(z)\,\mathcal{J}_{x,y}(t - z/c) + \theta(-z)\,\mathcal{J}_{x,y}(t + z/c)\right], \tag{14}$$

where $\theta(z)$ is the Heaviside step function with $\theta(z) = 1$ for $z \geq 0$ and zero elsewhere. The first term in Eq. (14) is the incoming wave. In the second line of Eq. (14), we see that after the encounter with the 2DEG two propagating waves are generated. One traveling in the propagation direction of the incoming pulse and one traveling in the opposite direction. The Heaviside functions ensure that the generated light propagates from the source located at $z = 0$. We assume that the spectrum is measured at a fixed observation point in the forward propagation direction. For the generated field at $z > 0$ we have

$$E_{x,y}^{(g)}(t - z/c) = -\frac{2\pi}{c}\mathcal{J}_{x,y}(t - z/c). \tag{15}$$