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Flux-driven quantum phase transitions in two-leg Kitaev ladder topological superconductor systems

H.Q. Wang, L.B. Shao, Y.M. Pan, R. Shen, L. Sheng, D.Y. Xing *

National Laboratory of Solid State Microstructures, School of Physics, and Collaborative Innovation Center of Advanced Microstructures, Nanjing University, Nanjing 210093, China

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ABSTRACT

We investigate a two-leg ladder topological superconductor system consisting of two parallel Kitaev chains with interchain coupling. It is found that either uniform or staggered fluxes threading through the ladder holes may change the ladder system from the BDI class in the Altland–Zirnbauer (AZ) classification to the D class. After explicitly calculating the topological Z and/or Z_2 indices and from the evolution of Majorana zero energy states (MZES), we obtain the flux-dependent phase diagrams, and find that quantum phase transitions between topologically distinct phases characterized by different number of MZES may happen by simply tuning the flux, which could be realized experimentally in ultracold systems.

1. Introduction

Recently, there has been a rapidly growing research on topological superconductors, partly because of their ability to host exotic quasiparticles known as Majorana fermions [1–4]. Majorana fermions are their own anti-particles and obey non-abelian exchange statistics, making them quite promising for fault-tolerant quantum computation. Up to now, various theoretical schemes have been proposed to realize condensed matter versions of Majorana fermions, such as the $\nu = 5/2$ fractional quantum Hall effect states [5], a strong topological insulator proximity-coupled to an ordinary *s*-wave superconductor [6], one-dimensional (1D) conventional semiconducting wires [7,8], and cold atomic gases [9,10].

Quasi-1D quantum wire systems have also attracted much interest [11–16], along with the experimental advancement in spin-orbit coupled nanowire systems [17–20]. For the quasi-1D multi-band nanowires with chiral symmetry, they are in the topological class BDI in the Altland–Zirnbauer (AZ) classification with an integer topological invariant Z which counts the number of zero-energy Majorana modes on a given end [11,21]. At the same time, there also exists a weak Z_2 topological invariant, which can only give the parity of the Z index but cannot distinguish between the presence and absence of the Majorana modes for even number of Z [11]. In a recent paper [12], the authors considered a ladder topological superconductor system, which was simulated by N 1D Kitaev chains [22] placed in parallel and coupled to each other, and investigated topological quantum transitions in the absence and presence of interchain superconducting pairing. It was found that the interchain pairing phase may change the system from class BDI characterized by the Z index to class D by the Z_2 index.

In this paper, we consider a two-leg Kitaev ladder topological superconductor system with interchain coupling, as shown in Fig. 1. Similar ladder systems have also been investigated in [23, 24]. However, instead of adding interchain superconductor pairing [12] or three-spin interaction [23], we apply uniform or staggered fluxes in all the ladder holes which is experimentally more simple and feasible. By use of symmetry analysis and calculation of the topological invariants Z_2 and/or Z, we find that applying uniform or staggered fluxes to the two-leg Kitaev ladder system will also break the time-reversal symmetry (TRS) of the system and thus play a crucial role in driving the quantum phase transition between the BDI class with multiple Majorana zero energy states (MZES) and the D class with only one or zero MZES. More importantly, in the TRS-broken D class, the number of MZES can be changed by varying either the interchain coupling or the inserted fluxes. We emphasize that the ladder's topological properties as well as the number of MZES can be manipulated by simply tuning the flux, which could be much easier to realize experimentally, e.g., in the cold atom systems by laser-assisted tunneling [25].

The rest of the paper is organized as follows. In Sec. 2, we analyze the two-leg Kitaev ladder topological superconductor system with uniform fluxes inside and in Sec. 3, we investigate the same

* Corresponding author.
E-mail addresses: nicholas2685@126.com (H.Q. Wang), dyxing@nju.edu.cn (D.Y. Xing).

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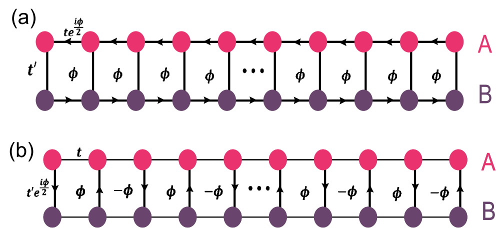

Fig. 1. (Color online) Schematic illustration of the two-leg Kitaev ladder system with (a) uniform and (b) staggered fluxes threading through each ladder hole. In the presence of these fluxes, the hopping phase is chosen as $e^{i\phi/2}$ ($e^{-i\phi/2}$) along the (reverse) direction of the arrow.

system but with staggered fluxes inside and make a brief discussion on the disappearance of pairs of MZES. Finally, a summary is given in Sec. 4.

2. Two-leg Kitaev ladder model with uniform fluxes inside

Consider a two-leg Kitaev ladder system shown in Fig. 1, where a couple of one-dimensional topological superconductor chains, denoted by A and B, respectively, are placed in parallel with interchain nearest-neighbor hopping amplitude t' [22,12]. For each chain, t is the intrachain nearest-neighbor hopping amplitude, Δ is the intrachain nearest-neighbor p-wave superconducting pairing amplitude, and μ is the chemical potential. Apply a series of identical fluxes ϕ to thread through all the ladder holes, as shown in Fig. 1(a), and adopt the translation-invariant gauge for which the effective intrachain hopping is $te^{i\phi/2}$ from right to left in subchain A and from left to right in subchain B. This specific gauge can be hopefully achieved through tuning the hopping phase by experimental methods such as laser-assisted tunneling [25]. Under this gauge, the lattice Hamiltonian can be written as

$$H = -\mu \sum_{n=1}^{N} \sum_{\tau=A,B} c^\dagger_{n\tau} c_{n\tau} - \sum_{n=1}^{N} (t' c^\dagger_{nA} c_{nB} + h.c.) - \sum_{n=1}^{N-1} \sum_{\tau=A,B} (t e^{i\phi\tau_z/2} c^\dagger_{n\tau} c_{n+1\tau} + \Delta c_{n\tau} c_{n+1\tau} + h.c.), \quad (1)$$

where $c^\dagger_n (c_n)$ is the creation (annihilation) operator and the pseudo-spin operator $\tau_z = 1$ (-1) for subchain A (B). Through the Fourier transformation $c_{n\tau} = (1/\sqrt{N}) \sum_k c_{k\tau} e^{ikn}$, we get the momentum-space Bogoliubov–de Gennes (BdG) Hamiltonian as

$$h(k) = \big(-\mu - 2t\cos k \cos\frac{\phi}{2}\big)\sigma_z\tau_0 + 2t \sin k \sin(\frac{\phi}{2})\sigma_0\tau_z - 2\Delta \sin k \sigma_y \tau_0 - t'\sigma_z\tau_x, \quad (2)$$

in the basis of $[c_{kA}, c_{kB}, c^\dagger_{-kA}, c^\dagger_{-kB}]^T$, where k belongs to the first Brillouin zone and the Pauli matrices τ_i and σ_i with $i = 0, x, y, z$ act in the A–B subchain space and particle-hole space, respectively.

To study the topological phase transition of this system, we first need to find the gap-closing points, since the closing and reopening process of the bulk gap is usually necessary for the transition between topologically distinct phases. Taking the spectrum of $h(k)$ to be zero, we get the relation $\mu = -2t\cos(\phi/2) \pm t'$ or $\mu = 2t\cos(\phi/2) \pm t'$, which may act as phase boundaries. The phase boundaries for $\phi = 0$ and $\phi \neq 0$ are plotted in the μ–t' plane of Figs. 2(a) and (b), respectively. Then we need to find a topological invariant to characterize each phase. We first study the case of $\phi \neq 0$. The Hamiltonian in Eq. (2) satisfies particle-hole symmetry (PHS) $\sigma_x h(k)^T \sigma_x = -h(-k)$ (triplet pairing) with the particle-hole reversal operator $\sigma_x K$ (K means complex conjugate), but it does not satisfy the TRS or chiral symmetry due to the presence of the second term on the right side of Eq. (2) provided that $\phi \neq 0$. As a result, the BdG Hamiltonian for nonzero ϕ falls into class D with a Z_2 topological invariant in the 1D system [21]. For any BdG Hamiltonian that satisfies the above PHS, a Z_2 index can be defined as Q=sgn[Pf$\{h(k=\pi)\sigma_x\}$/Pf$\{h(k=0)\sigma_x\}$], where $k = 0$ (π) is the particle-hole symmetric momentum [11] and Pf means Pfaffian. In our system, the second term of Eq. (2) vanishes at $k = 0$ or π, and the calculation for Q can be simplified as [11]

$$Q = \mathrm{sgn}[\frac{\mathrm{Det}\{(-\mu + 2t\cos\frac{\phi}{2})\tau_0 - t'\tau_x\}}{\mathrm{Det}\{(-\mu - 2t\cos\frac{\phi}{2})\tau_0 - t'\tau_x\}}] = \begin{cases} -1 & \text{for } \left||2t\cos\frac{\phi}{2}| - t'\right| < |\mu| < |2t\cos\frac{\phi}{2}| + t', \\ 1 & \text{for } |\mu| > |2t\cos\frac{\phi}{2}| + t' \text{ or } |\mu| < \left||2t\cos\frac{\phi}{2}| - t'\right|. \end{cases} \quad (3)$$

In the case of $\phi \neq 0$, Q acts as the Z_2 topological invariant, with $Q = -1$ indicating the nontrivial phase with one $(N = 1)$ MZES and $Q = 1$ indicating the trivial phase without MZES $(N = 0)$. It is easily seen from Eq. (3) that the phase boundaries across which Q changes its sign are exactly consistent with those calculated from the gap closing conditions. As shown in Fig. 2(b), three boundary lines divide the μ–t' plane into four regions; the shadow region of $Q = -1$ is the topological phase and others $(Q = 1)$ are trivial. We wish to point out here that in this phase diagram, the line of $t' = 0$ is special and needs to be discussed separately, where the two subchains get decoupled and can be treated as two copies of 1D TRS-broken Kitaev model with the Z_2 index of each subchain given by $Q = \mathrm{sgn}\big[(\mu - 2t\cos(\phi/2))/(\mu + 2t\cos(\phi/2))\big]$. For $|\mu| < 2t|\cos(\phi/2)|$ and $t' = 0$, since each subchain has a contribution of $N = 1$ with $Q = -1$, we have $N = 2$ and there are two MZES in the red section of line $t' = 0$.

Second, we focus on the case of $\phi = 0$ (no flux). In this case, since the second term in Eq. (2) vanishes, apart from the PHS, the Hamiltonian also satisfies TRS, $h(k)^* = h(-k)$, and chiral symmetry, $\sigma_x h(k)\sigma_x = -h(k)$, with the chiral operator σ_x constructed by multiplying particle-hole operator $\sigma_x K$ and time-reversal operator K. It thus belongs to class BDI which is characterized by a Z topological invariant in the 1D system [21]. Based on the chiral symmetry, the BdG Hamiltonian can be brought into an off-diagonal form under a rotation in the particle-hole space by the unitary operator $U = e^{-i(\pi/4)\sigma_y}$

$$h(k)'_{\phi=0} = U h(k)_{\phi=0} U^\dagger = \begin{pmatrix} 0 & q(k) \\ q(-k)^T & 0 \end{pmatrix}, \quad (4)$$

with $q(k) = (-\mu - 2t\cos k + 2i\Delta\sin k)\tau_0 - t'\tau_x$, and then another topological number, called the winding number, can be defined as [11,12]

$$W \equiv \frac{-i}{2\pi}\int_0^{2\pi} dk\partial_k \ln\det q(k) = \frac{-i}{2\pi}\mathrm{tr}\int_0^{2\pi} dk\partial_k \ln q(k) = \frac{-i}{2\pi}\sum_{n=1,2}\int_0^{2\pi} dk\partial_k \ln\lambda_n(k), = \begin{cases} -2 & \text{for } |\mu| < |2t - t'|, \\ -1 & \text{for } |2t - t'| < |\mu| < 2t + t', \\ 0 & \text{for } |\mu| > 2t + t'. \end{cases} \quad (5)$$

where

$$\lambda_{1,2}(k) = -\mu - 2t\cos k + 2i\Delta\sin k \pm t' \quad (6)$$