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Localization length fluctuation in randomly layered media

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A B S T R A C T

Localization properties of the two-component randomly layered media (RLM) are studied in detail both analytically and numerically. The localization length is found fluctuating around the analytical result obtained under the high-frequency limit. The fluctuation amplitude approaches zero with the increasing of disorder, which is characterized by the distribution width of random thickness. It is also found that the localization length over the mean thickness periodically varies with the distribution center of random thickness. For the multi-component RLM structure, the arrangement of material must be considered.

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1. Introduction

Localization, known as the absence of the transport wave, is one of the most important properties of disordered media [1–5]. The one-dimensional disordered structure typically refers to the randomly layered media (RLM), which is a stack of dielectric films with randomly varying thickness. RLM has been applied in realizing the necklace states [6], the optical bistability [7–9], the broadband reflector [10,11], and so on [12–17]. The theoretical study mainly focuses on the two-component RLM with high-frequency limit (strong disorder), which means that the distribution width is much larger than the wavelength [18]. However, as far as we know, analytical discussion about localization properties of RLM with the weak disorder has not been studied yet, which could be helpful to understand the behavior of the non-ideal photonic crystal and optimizing the parameters of RLM to reach enough localization strength.

The localization property of RLM is characterized by the inverse localization length, l_{loc}^{-1}, which means that the field amplitude decreases by e^{-1} per l_{loc} on average. In general, the localization length is calculated from statistical results of numerical simulation or experiment. The analytical expression had been derived only when both the approximation of the zero-energy flux and the complete phase stochastization were satisfied [18]. The RLM has the transmissivity $T \sim \exp(-2l_{\text{loc}}^{-1}L)$ in statistics (L is the total length of RLM). Transmission energy flux exponentially decreases with L increasing. Hence, an RLM structure with L large enough acts just as a broadband perfect reflector, where only the standing wave can be found. It means that the former assumption of the zero-energy flux approximation would always establish in the case of considering a large enough RLM structures [10,11].

The phase of each layer is considered to be the main variable of profiling the field in RLM. The latter assumption of the complete phase stochastization means the phase uniformly distributed on $(0, 2\pi)$. It only stands for the high-frequency limit (or strong disorder, i.e., the distribution width of the random thickness is much larger than the wavelength). Finally, the integral form of localization length can be simplified to a simple and full analytical expression (see Ref. [18]). However, if the distribution width of the random thickness is comparable to the wavelength, i.e., the weak disorder condition, the assumption of complete phase stochastization is no longer applicable. The phase distributions are related to each other and become non-uniform distributions. The inverse localization length is formed in an integration expression finally.

In this work, the localization properties of the two-component RLM are studied both analytically and numerically. In section 2, an iteration method is raised to obtain the non-uniform phase distribution in the case of weak disorder when the high-frequency limit is unsatisfied. An analytical expression of the localization length is derived in the integral form. In section 3, the numerical results are presented. It is found that the localization length is determined by the refractive indexes of each component and the distribution

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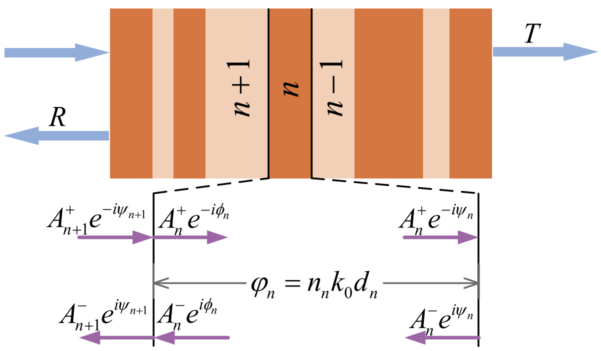

Fig. 1. (Color online.) The schematic diagram of the randomly layered media.

characteristic of the random thickness, which is described by the distribution center and distribution width. We also discuss the influences of the layer arrangement for three-component RLM at the end.

2. Structure and method

The two-component RLM consists of dielectric layers of alternating refractive index ε_A and ε_B as shown in Fig. 1. We use the phase defined as $\varphi_n = k_0 n_n d_n$ to represent the thickness, instead of the real thickness d_n. The refractive index of layer n is $n_n = (\varepsilon_n)^{1/2}$ and $k_0 = 2\pi/\lambda_0$ is the vacuum propagation constant at wavelength λ_0. For simplicity, we just consider the special condition that the random thicknesses of all the layers uniformly distribute in the same interval of $\varphi_n \in (\Phi_0 - \Phi_1, \Phi_0 + \Phi_1)$, $n = 1, 2, \cdots, N$. For the condition of normal incidence, it is possible that we use three parameters, A_n^+, A_n^- and ϕ_n (or ψ_n), to characterize the electric field in layer n as shown in Fig. 1. By regarding the structure as a part of a large RLM, the zero flux approximation should be established [10,11], which means $A_n^+ = A_n^- = A_n$.

In each layer, the amplitude has no change. The two phases of ϕ_n and ψ_n are related by

$$\phi_n = \varphi_n + \psi_n. \tag{1}$$

At the boundary, the incident field $A_{n+1}e^{-i\psi_{n+1}}$ falls to $A_{n+1}e^{-i\psi_{n+1}}t_{n+1,n}$ after the interface of $n+1 \to n$. $t_{n+1,n}$ represents the transmission coefficient at the interface of $n+1 \to n$. $t_{n+1,n}$. And then, it will be amplified by the cavity resonance with the amplification coefficient $L_n(\phi_n)$:

$$L_{n+1}(\phi_n) = \frac{1}{1 - r_{n,n+1}e^{2i\phi_n}}, \tag{2}$$

where $r_{n,n+1}$ is the reflection coefficient at the interface $n \to n+1$. Hence, $A_{n+1}e^{-i\psi_{n+1}}$ is relative to $A_n e^{-i\phi_n}$ by

$$A_n e^{-i\phi_n} = A_{n+1}e^{-i\psi_{n+1}}t_{n+1,n}L_{n+1}(\phi_n), \tag{3}$$

Eq. (2) and Eq. (3) can be simplified to

$$L_{n+1}(\phi_n)| = \frac{\eta_{n+1} + \eta_n}{2\sqrt{\eta_{n+1}{}^2\cos^2\phi_n + \eta_n{}^2\sin^2\phi_n}} \tag{4}$$

$$\psi_{n+1}(\phi_n) = \arg(\eta_{n+1}\cos\phi_n + i\eta_n \sin\phi_n), \tag{5}$$

where the $\eta_n = (\varepsilon_0\varepsilon_n/\mu_0)^{-1/2}$ is the wave admittance of layer n. The transmissivity at the boundary $n+1 \to n$ is

$$T_{n+1,n}(\phi_n) = \left|A_n/A_{n+1}\right|^2 \eta_n/\eta_{n+1}. \tag{6}$$

For the whole N-layer RLM, we obtain

$$T_{N,1} = T_{N,N-1}(\phi_{N-1})T_{N-1,N-2}(\phi_{N-2})\cdots T_{2,1}(\phi_1). \tag{7}$$

ϕ_n on each layer can be obtained from the iteration relation of Eq. (1) and Eq. (5) with an initial phase ϕ_1.

It is found that the choice of the initial phase ϕ_1 barely affects the final result of $T_{N,1}$ due to the localization property of the RLM structure. According to Eq. (1) and Eq. (5), a small variation of $\delta\phi_1$ spreading from layer 1 to layer 2 becomes

$$\delta\phi_2 = \partial_{\phi_1}\phi_2\delta\phi_1 = T_{2,1}\delta\phi_1. \tag{8}$$

Eq. (8) is also an iteration formula and $\delta\phi_n = T_{n,1}\delta\phi_1$ is obtained. According to the localization property of RLM, $T_{n,1}$ is an exponentially small value when n is large enough. Therefore, the small variation of $\delta\phi_1$ barely affects the phase on layer n. It means that no matter what value we choose as the initial phase ϕ_1, $\{\phi_n\}$ will be rapidly convergent to the same sequence, which is determined by the structure.

The localization property is characterized by the inverse localization length l_{loc}^{-1}. It has the meaning that the field amplitude reduces to e^{-1} after propagating a length of l_{loc} on average, and the transmissivity exponentially decreases with total length L increasing. That is $\langle T_{N,1}\rangle = \exp[-2l_{\mathrm{loc}}^{-1}(N-1)s_0]$, where $s_0 = (\varepsilon_A^{-1/2} + \varepsilon_B^{-1/2})\lambda_0\Phi_0/4\pi = \langle L\rangle/(N-1)$ is the mean thickness of each layer. $\langle T_{N,1}\rangle$ with fixed N is a self-averaging quality, which means the $T_{N,1}$ of any large enough RLM coincides with its mean value with the exponential accuracy [4,18]. So the $T_{N,1}$ of a large enough RLM can take the place of the statistical $\langle T_{N,1}\rangle$ of a large amount of RLM structures with an acceptable error. Because the terms in the product of Eq. (7) are commutative, the only thing we need is the distribution of $T_{n+1,n}(\phi_n)$, which is determined by the distribution property of the sequence $\{\phi_n\}$. All the odd (even) layers are equivalent for the two-component RLM as shown in Fig. 1. It means that all the $\{\phi_n\}$ label odd (even) distribute the same. We label the odd (even) interface of A → B (B → A) to I (II) and the two probability density functions of $\{\phi_n\}$ can be expressed as $g_{\mathrm{I}}(\phi)$ and $g_{\mathrm{II}}(\phi)$, respectively. Notice that there should be no differences between the phase distributions obtained from the statistics of a large RLM or a large amount of RLM structures. In other words, if we use the probability density function $g_n(\phi_n)$ to describe the phase distribution of layer n, then $g_{2n+1}(\phi_{2n+1}) = g_{\mathrm{I}}(\phi)$ and $g_{2n}(\phi_{2n}) = g_{\mathrm{II}}(\phi)$ will establish for odd and even layers, respectively.

The probability density function of phase ψ_n and $\phi_n \in (-\pi/2, \pi/2)$ are $f_n(\psi_n)$ and $g_n(\phi_n)$, respectively. The relationship between $g_n(\phi_n)$ and $g_{n+1}(\phi_{n+1})$ can be obtained from the iteration relation of Eq. (1) and Eq. (5). Eq. (5) gives ψ_n as a function of ϕ_{n-1}. With its inverse function of $\phi_{n-1}(\psi_n)$, we have the iteration relation of $f_n(\psi_n) = g_{n-1}[\phi_{n-1}(\psi_n)]/\psi_n'[\phi_{n-1}(\psi_n)]$, which could be simplified to

$$f_n(\psi_n) = \frac{\eta_{n-1}\eta_n}{\eta_{n-1}^2\cos^2\psi_n + \eta_n^2\sin^2\psi_n} \\ g_{n-1}[\arg(\eta_{n-1}\cos\psi_n + i\eta_n\sin\psi_n)]. \tag{9}$$

$f_n(\psi_n)$ and $g_n(\phi_n)$ have a periodicity of π. As probability density functions, $\int_{-\pi/2}^{\pi/2} d\psi f_n(\psi) = \int_{-\pi/2}^{\pi/2} d\phi g_{n-1}(\phi) = 1$ must be satisfied. The thickness φ_n homogeneously distributes in the interval of $(\Phi_0 - \Phi_1, \Phi_0 + \Phi_1)$, that is the probability density functions of the uniform distribution of φ_n is

$$h_n(\varphi_n) = \frac{1}{2\Phi_1}\mathrm{rect}(\frac{\varphi_n - \Phi_0}{\Phi_1}). \tag{10}$$

According to Eq. (1), the probability density function of $g_{n-1}(\phi_{n-1})$ should be the convolution of $h_{n-1}(\varphi_{n-1})$ and $f_{n-1}(\psi_{n-1})$:

$$g_{n-1}(\phi_{n-1}) = f_{n-1}(\phi_{n-1}) * h_{n-1}(\phi_{n-1}). \tag{11}$$

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