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Two kinds of magnetic gauge potentials due to coherent effect in two-gap superconductor

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ABSTRACT

Two-component Ginzburg–Landau model with one magnetic gauge potential can be used to describe the physical properties of two-gap superconductor. When the order parameters in two-gap superconductor have different phases, the gauge invariance will be destroyed. In order to preserve gauge invariance, two kinds of gauge potentials must be introduced. For seeking the origins of two kinds of gauge potentials, one suggests two kinds of order parameters are in the coherent state. Therefore, two different gauge potentials and masses of the order parameters arise through deducing the super-current of the coherent state. As a result, two different gauge potentials lead to different magnetic fields at the zero points of the order parameters. In other places, the gauge potentials have no contributions to the magnetic field. Moreover, the topological properties of two different gauge potentials are discussed in detail.

Introduction

For the reason that the two-component superconductor exhibits some novel properties, two-component superconductor [1–4] has inspired great interests since high temperature superconductor MgB_2 was observed in the experiments [5–7]. For example, there are composite vortices with fractional magnetic flux [8–10]; the two-component superconductor is very different from the type I and type II superconductor. In Refs. [11,12], the authors call it type 1.5 superconductor. The different electronic charges carried by the condensates are studied by using of Ginzburg–Landau theory [13, 14]. In general, all these problems can be described by the multi-band Ginzburg–Landau theory. In this model, two kinds of order parameters are introduced to describe the two kinds of condensates, which are coupled to same gauge potential. A problem immediately is raised, which is pointed out in [15,16], the Ginzburg–Landau theory can not preserve gauge invariance when the order parameters have different phases. To overcome this difficulty, it is necessary to build a $U(1) \times U(1)$ symmetry including two kinds of gauge potentials [16]. These two kinds of gauge potentials couple to two kinds of order parameters respectively. Therefore, a general non-interacting multi-band system is introduced. But the important and interest problem is what is the origins of two kinds of gauge potentials. In this letter, we propose the reason leading to two kinds of gauge potentials is the coherent effect between the order parameters. From Ginzburg–Landau model with one gauge potential and considering the coherent effect, we will deduce the non-interacting multi-band model with two kinds of gauge potentials.

This paper is arranged as follows: In section 1, the Ginzburg–Landau theory is presented and reviewed. In this case, the order parameters have two different kinds of phases are assumed, we show the $U(1)$ local gauge invariance can not be preserved under $U(1)$ local transformation. Especially, the super-current in this model is presented in detail. In section 2, the order parameters are suggested to be in the coherent state, the super-current of the coherent state is given by using reduced density matrix theory. Then, two kinds of magnetic gauge fields are achieved naturally; we also reveal that the masses of two kinds of order parameters are different. The non-interacting multi-band model can be deduced from the super-current. In section 3, the topological properties of the gauge potential are studied by using ψ-mapping topological current theory in detail.

1. Ginzburg–Landau model with one gauge potential

In general, the phenomenological description of the two-gap superconductor is given as

$$H = \sum_{i=1}^{2} \frac{1}{2m_i} (\nabla + 2ie\mathbf{A}) \psi_i^* (\nabla - 2ie\mathbf{A}) \psi_i + V(\psi) + \frac{1}{2\mu_0} (\nabla \times \mathbf{A}), \quad (1)$$

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where potential $V(\psi) = \alpha |\psi_i|^2 + \beta |\psi_i|^4$. ψ_i are the order parameters describing the condensates in the two-gap superconductor. It is well known as two-component Ginzburg–Landau model. When the order parameters have same phase there is one gauge potential $\mathbf{A}$ in the model. The gauge invariance is preserved under $U(1)$ gauge transformations. Normally, the $U(1)$ gauge transformations are given as

$$\psi_i(x) \to \psi_i'(x) \equiv e^{-2ie\theta(x)}\psi_i(x),$$
$$\mathbf{A}(x) \to \mathbf{A}'(x) \equiv \mathbf{A}(x) - \nabla\theta(x). \tag{2}$$

From (1) and by using $\mathbf{J} = \frac{\delta H}{\delta \mathbf{A}}$, the super-current can be archived

$$\mathbf{J} = \frac{ie}{m_1}\left(\psi_1^*\nabla\psi_1 - \psi_1\nabla\psi_1^*\right) + \frac{ie}{m_2}\left(\psi_2^*\nabla\psi_2 - \psi_2\nabla\psi_2^*\right) + 4e^2\left(\frac{|\psi_1|^2}{m_1} + \frac{|\psi_2|^2}{m_2}\right)\mathbf{A}. \tag{3}$$

This super-current also can be rewritten as

$$\mathbf{J} = \frac{ie}{m_1}\left[\psi_1^*(\nabla - 2ie\mathbf{A})\psi_1 - \psi_1(\nabla + 2ie\mathbf{A})\psi_1^*\right] + \frac{ie}{m_2}\left[\psi_2^*(\nabla - 2ie\mathbf{A})\psi_2 - \psi_2(\nabla + 2ie\mathbf{A})\psi_2^*\right]. \tag{4}$$

Moreover, this form of the super-current shows that the two kinds of order parameters are not in the coherent state. The only one gauge potential will lead to serious problem that is the $U(1)$ local gauge invariance can not be preserved when two kinds of order parameters have different phases. Let us consider the order parameters and the gauge potential transform as

$$\psi_i(x) \to \psi_i'(x) \equiv e^{-2ie\theta_i(x)}\psi_i(x), \tag{5}$$
$$\mathbf{A}(x) \to \mathbf{A}'(x) \equiv \mathbf{A}(x) - \nabla\sigma(x). \tag{6}$$

Put these relation into (1), the term in the kinetic destroys $U(1)$ local gauge invariance, it is

$$2ie\left(\nabla\theta_i(x) - \nabla\sigma(x)\right). \tag{7}$$

In order to preserve the gauge invariance in the $U(1)$ local gauge transformations (5), two kinds of gauge potentials must be introduced in the Ginzburg–Landau model. In the following, one will find how the two kinds of gauge potentials depend on the coherent effect of the order parameters.

2. Coherent effect in the order parameters and its application in two-component Ginzburg–Landau model

In order to find why there are two kinds of gauge potentials in two-gap superconductor, we suggest the order parameters are in the coherent state:

$$|\psi\rangle = C_1|\psi_1\rangle + C_2|\psi_2\rangle, \tag{8}$$

where $|C_1|^2 + |C_2|^2 = 1$. Recall the reduced density matrix theory in Ref. [17], these yields

$$\psi = C_1\psi_1 + C_2\psi_2. \tag{9}$$

By considering (4), the super-current of the coherent state is

$$\mathbf{J} = \frac{ie}{m}\left(\psi^*(\nabla - 2ie\mathbf{A})\psi\right) - \psi(\nabla + 2ie\mathbf{A})\psi^*. \tag{10}$$

Then put (9) into (10), we have

$$\begin{aligned}\mathbf{J} = {} & \frac{ie}{m}C_1C_1^*\left(\psi_1^*\nabla\psi_1 - \psi_1\nabla\psi_1^*\right) + \frac{ie}{m}C_2C_2^*\left(\psi_2^*\nabla\psi_2 - \psi_2\nabla\psi_2^*\right) \\ & + \frac{ie}{m}\big(C_1C_2^*\psi_1\nabla\psi_2^* - C_1^*C_2\psi_1^*\nabla\psi_2 \\ & + C_1^*C_2\psi_2\nabla\psi_1^* - C_1C_2^*\psi_2^*\nabla\psi_1\big) \\ & + 4e^2\left(\frac{C_1C_1^*|\psi_1|^2}{m} + \frac{C_2C_2^*|\psi_2|^2}{m}\right)\mathbf{A} \\ & + 4e^2\left(\frac{C_1^*C_2\psi_1^*\psi_2}{m} + \frac{C_1C_2^*\psi_1\psi_2^*}{m}\right)\mathbf{A}.\end{aligned} \tag{11}$$

The third term in the super-current (11) is denoted as $\tilde{\mathbf{j}}_3$

$$\tilde{\mathbf{j}}_3 = \frac{ie}{m}\big(C_1C_2^*\psi_1\nabla\psi_2^* - C_1^*C_2\psi_1^*\nabla\psi_2 + C_1^*C_2\psi_2\nabla\psi_1^* - C_1C_2^*\psi_2^*\nabla\psi_1\big), \tag{12}$$

which is

$$\begin{aligned}\tilde{\mathbf{j}}_3 = {} & \frac{ie}{m}\Big(C_1^*C_2\psi_1^*\psi_2\frac{\psi_1\nabla\psi_1^*}{\psi_1^*\psi_1} - C_1C_2^*\psi_1\psi_2^*\frac{\psi_1^*\nabla\psi_1}{\psi_1^*\psi_1} \\ & + C_1C_2^*\psi_2^*\psi_1\frac{\psi_2\nabla\psi_2^*}{\psi_2^*\psi_2} - C_1^*C_2\psi_2\psi_1^*\frac{\psi_2^*\nabla\psi_2}{\psi_2^*\psi_2}\Big).\end{aligned} \tag{13}$$

Let us define

$$C_1C_2^*\psi_1\psi_2^* = \Lambda_1 + i\Lambda_2,$$
$$C_1^*C_2\psi_1^*\psi_2 = \Lambda_1 - i\Lambda_2. \tag{14}$$

According to reduced density matrix theory [17,18], the parameter Λ_1 is used to represent the interference effect of the coherent state. When $\Lambda_1 = 0$, the interference effect disappears. So, one calls the parameter Λ_1 as coherent parameter. By using the relations (14), one has

$$\begin{aligned}\tilde{\mathbf{j}}_3 = {} & \frac{ie}{m}\left[\Lambda_1\frac{\psi_1^*\nabla\psi_1 - \psi_1\nabla\psi_1^*}{\psi_1^*\psi_1} + \Lambda_1\frac{\psi_2^*\nabla\psi_2 - \psi_2\nabla\psi_2^*}{\psi_2^*\psi_2}\right] \\ & - \frac{e}{m}\Lambda_2\left[\left(\frac{\psi_1^*\nabla\psi_1}{\psi_1^*\psi_1} + \frac{\psi_1\nabla\psi_1^*}{\psi_1^*\psi_1}\right) + \left(\frac{\psi_2^*\nabla\psi_2}{\psi_2^*\psi_2} + \frac{\psi_2\nabla\psi_2^*}{\psi_2^*\psi_2}\right)\right].\end{aligned} \tag{15}$$

For simply, one writes the densities of the order parameters are

$$\rho_i = \psi_i^*\psi_i \quad i = 1, 2. \tag{16}$$

Then the super-current is

$$\tilde{\mathbf{j}}_3 = \frac{ie}{m}\frac{\Lambda_1}{\rho_1}\left(\psi_1^*\nabla\psi_1 - \psi_1\nabla\psi_1^*\right) + \frac{ie}{m}\frac{\Lambda_1}{\rho_2}\left(\psi_2^*\nabla\psi_2 - \psi_2\nabla\psi_2^*\right) - \frac{e}{m}\Lambda_2\left(\nabla\ln\rho_1 + \nabla\ln\rho_2\right). \tag{17}$$

In this expression, one finds the terms $\nabla\ln\rho_i$ $(i = 1, 2)$ are vectors fields. It is should be noted the magnetic gauge potential is also vector field. Then the vectors $\nabla\ln\rho_i$ can be added into the magnetic gauge potential, the direct result is the magnetic gauge potentials have different values. In general, because the vectors satisfy Clairaut's theorem:

$$\nabla\times\nabla\ln\rho_1 = 0,$$
$$\nabla\times\nabla\ln\rho_2 = 0, \tag{18}$$

the different magnetic gauge potentials do not lead to different magnetic fields. However, in some special points, such as the zero points of the order parameters, the curls of the vectors do not