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The influence of non-Gaussian noise on the accuracy of parameter estimation

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ABSTRACT

We analyze the influence of non-Gaussian noise on the accuracy of the parameter estimation. Specially, we estimate the intensity of magnetic field by adopting a spin-1/2 system which is influenced by random telegraph noise and colored noise with $1/f^{\alpha}$ spectrum. We find that, for the random telegraph noise case, both weak coupling and strong coupling conditions are suitable for the parameter estimation. For the colored noise case, the accuracy of the parameter estimation decreases with the parameter α firstly and then increases with it. We also give physical explanations for these phenomena.

1. Introduction

Parameter estimation, serving as one of vital missions in the quantum metrology [1–4], is of great significance in both quantum physics and quantum information processing field. Recently, there have been a lot of works focusing on the parameter estimation in various situations [5–19]. The main task of parameter estimation is to find the value of an unknown parameter included in the system by using an optimal measurement. Quantum parameter estimation can be thought of as comprising three steps: (i) a preparation step in which a quantum system as a probe is initially prepared in the state $\rho(0)$, (ii) an encoding step in which the parameter is encoded onto the probe and (iii) a measurement step in which the probe is measured to get the information about the parameter. Its primary aim is to measure the parameter as accurately as possible. Quantum Fisher information [1,2,20] which characterizes the accuracy of the parameter estimation, plays an important role in the parameter estimation process. It provides the upper limit on the precision of the estimation, which is expressed as the Cramér–Rao inequality. Therefore, it is important to investigate the behaviors of the quantum Fisher information and then how to increase the value of it becomes our key problem to be solved if we want to get the maximum information about the parameter to be estimated.

However, it is generally impossible to isolate a particular quantum system in which we are interested from its surroundings. Hence the dynamics of the relevant quantum system should be described by open quantum theory [21,22] in order to faithfully represent its dynamical evolution. Nowadays, quantum Fisher information for the parameter estimation in the open systems has also been widely studied [19,23–31].

Parameter estimation under the influence of quantum noise has been investigated by many authors [26–30]. It has been shown that the collisional dephasing dramatically diminishes the precision of the phase estimation with the Ramsey interferometry [29]. The non-Markovian effect on the dynamics of the quantum Fisher information has been studied [19,30]. It has been proved that quantum noise induced by interaction of a qubit with a quantum bath can be mimicked by a random classical noise acting upon the qubit without the need for a bath and this random unitary evolution is equivalent to the quantum case [32]. Due to the central limit theorem, it is always assumed that the fluctuations of the noise obey Gaussian statistics. Quantum parameter estimation under the Gaussian classical noise has been investigated [33,34].

In solid-state realization of qubits, material-specific fluctuations typically induce the major contribution to the intrinsic noise. Experiments over the years have shown that the noise the quantum device demonstrates has a $1/f^{\alpha}$ spectrum [35,36]. This suggests that the environment that destroys the phase coherence of the qubit can be regarded as a system of two-state fluctuators, which experience random hops between their states. In many important cases the noise produced by the fluctuators is non-Gaussian [37]. More recently, the classical noise exhibiting non-Gaussian fluctuations has been studied and it has been shown that noise with a spectrum $1/f^{\alpha}$ is generally non-Markovian and the non-Markovianity of colored noise decreases when the number of

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fluctuators realizing the noise increases [38]. The characteristic parameters of non-Gaussian noise [39] and Gaussian noise [40] are estimated by using a simple quantum probe. This is an ingenious method to estimate the characteristic parameters of noise itself.

In this paper, we focus on the influence of non-Gaussian noise on the accuracy of parameter estimation. Particularly, we adopt a spin-1/2 system to estimate the intensity of a magnetic field and analyze the dynamics of the quantum Fisher information under two typical models: the random telegraph noise and the colored noise. We find that the quantum Fisher information is greatly influenced by the non-Gaussian noise. More specially, on one hand, the smaller switching rate γ in the random telegraph noise case and the lower spectrum frequency in the colored noise case correspond to different oscillation phenomena of the quantum Fisher information. And the smaller the value of γ and α, the greater the maximum value of the quantum Fisher information. On the other hand, for the larger switching rate γ and the higher spectrum frequency, the quantum Fisher information increases at first and then reduces to zero as time evolves. Also the larger the value of γ and α, the larger the maximum value of the quantum Fisher information, thus the accuracy of the parameter estimation can arise in these conditions.

The rest of the paper is organized as follows. In Sec. 2, we review the quantum Fisher information and show its analytic form in the Bloch representation. In Sec. 3, we give the models of the non-Gaussian noise, which are characterized by random telegraph noise and colored noise. In Sec. 4 and Sec. 5, we study the dynamics of the quantum Fisher information under the non-Gaussian noise and analyze the physical reasons for its behaviors in detail. Finally we conclude our results in Sec. 6.

2. Quantum fisher information

According to the quantum estimation theory, the ultimate achievable precision $\Delta\omega$, where ω is the parameter to be estimated, is limited by the quantum Cramér–Rao inequality

$$\Delta\omega \geq \frac{1}{\sqrt{MF_\omega}}, \tag{1}$$

where M is the total number of independent trials performed on the same system. F_ω is the quantum Fisher information defined by

$$F_\omega = \mathrm{Tr}[\rho_\omega L_\omega^2], \tag{2}$$

with respect to the state ρ_ω. While L_ω is the symmetric logarithmic derivative given by

$$\frac{\partial\rho(\omega)}{\partial\omega} = \frac{1}{2}[L_\omega\rho_\omega + \rho_\omega L_\omega]. \tag{3}$$

Particularly, for the qubit system, F_ω can be explicitly given by [41]

$$F_\omega = \mathrm{Tr}(\partial_\omega\rho_\omega)^2 + \frac{1}{\det\rho_\omega}\mathrm{Tr}(\rho_\omega\partial_\omega\rho_\omega). \tag{4}$$

Furthermore, single qubit can be described as

$$\rho_\omega = \frac{1}{2}[\hat{\mathbf{1}} + \vec{a}(\omega)\cdot\vec{\sigma}], \tag{5}$$

with $|\vec{a}(\omega)| \leq 1$ in Bloch representation. Hence, it immediately yields the quantum Fisher information in terms of $\vec{a}(\omega)$.

If the qubit state is pure, the density matrix satisfies $\rho^2 = \rho$ and the Bloch vector satisfies the condition $|\vec{a}(\omega)| = 1$. Then we can obtain F_ω for the pure state as

$$F_\omega = |\partial_\omega\vec{a}(\omega)|^2. \tag{6}$$

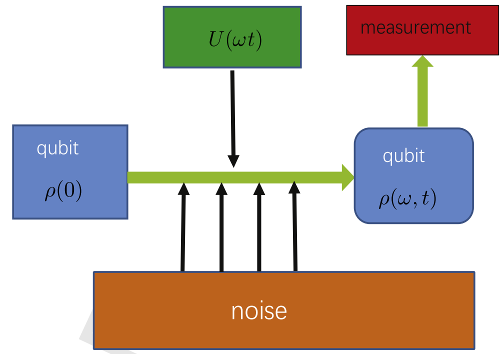

Fig. 1. (Color online.) A presentation of the frequency estimation. The qubit is initially in state $\rho(0)$. Then the qubit is subjected to the parameter encode process and meanwhile it interacts with the noise, with ω being the parameter to be measured. Finally the frequency estimation is extracted from the $\rho(\omega, t)$ of the qubit.

For the mixed state, $|\vec{a}(\omega)| < 1$, it has been shown that [29] in the Bloch representation F_ω can represent as

$$F_\omega = |\partial_\omega\vec{a}(\omega)|^2 + \frac{[\vec{a}(\omega)\cdot\partial_\omega\vec{a}(\omega)]^2}{1 - |\vec{a}(\omega)|^2}. \tag{7}$$

It becomes much simpler to use Eq. (7) to study the dynamics of the quantum Fisher information under different noises, including the Gaussian noise and non-Gaussian noise, specially random telegraph noise and colored noise.

3. Model

As shown in Fig. 1, the estimation process is decomposed into three procedures: Firstly, as the preparation step, we choose the initial input state as $\rho(0)$ ensuring that the quantum Fisher information of the output state reaches its maximum value. Then make the system subject to a magnetic field, meanwhile the qubit is inevitably influenced by the noise. Finally, we perform the proper measurements and construct the estimation for the unknown parameter.

Here we consider a qubit system interacting with the magnetic field and the environment, which is described by the Hamiltonian

$$H(t) = \frac{1}{2}\mu\omega\sigma_z + \frac{1}{2}\nu c(t)\sigma_z. \tag{8}$$

Hamiltonian (8) represents a class of models of open quantum systems that describe a pure dephasing process. In Eq. (8), ω is the unknown parameter to be estimated, e.g., it can be the magnetic field. μ and ν are the coupling constants between the qubit and the magnetic field, noise respectively. σ_z is the Pauli matrix. $c(t)$ denotes a stochastic process of the environment noise and it may has different expressions corresponding to different kinds of noise.

The density matrix for the qubit can be obtained by taking ensemble average over the noise $c(t)$:

$$\rho(t) = \langle\rho_{st}(t)\rangle, \tag{9}$$

where $\langle\ldots\rangle$ stands for ensemble average and the statistical density operator $\rho_{st}(t)$ is given by

$$\rho_{st}(t) = U(t)\rho(0)U^\dagger(t), \tag{10}$$

where $\rho(0)$ is the initial state of the probe system. The unitary operator $U(t)$ can be written as

$$U(t) = \exp\left[-i\int dt' H(t')\right]. \tag{11}$$

Then the reduced density matrix of the qubit dynamics with the time is obtained by