Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Sherrington–Kirkpatrick glassy-phase of random Josephson coupled Bose–Einstein condensates in wood-pile geometry

T.K. Kopeć [a,b,*], M.M. Musiał [c]

[a] *Institute for Low Temperature and Structure Research, Polish Academy of Sciences, PO. Box 1410, 50-950 Wrocław 2, Poland*
[b] *Institute of Theoretical Physics, University of Wrocław, Pl. Maksa Borna 9, 50-204 Wrocław, Poland*
[c] *Faculty of Fundamental Problems of Technology, Wrocław University of Technology, Wybrzeze Wyspianskiego St. 27, 50-370 Wrocław, Poland*

ARTICLE INFO

Article history:
Received 5 October 2016
Received in revised form 16 November 2016
Accepted 16 November 2016
Available online xxxx
Communicated by V.A. Markel

Keywords:
Bose condensation
Optical lattices
Josephson effect
Phase transition
Disorder
Glassy phase

ABSTRACT

We study a transition to a glassy phase of neutral atoms trapped in optical lattice, where the system is realized as the array of individual Bose–Einstein condensates of N elongated vertical and horizontal N rods in a wood-pile form coupled via the random Josephson tunneling. In this geometry every horizontal (vertical) rod of a condensate is linked to its vertical (horizontal) counterpart, so that the number of nearest neighbors z of a given rod in this system is $z = N$, implying that the system is fully connected. This together with randomness forms a prerequisite of the Sherrington–Kirkpatrick model for $N \to \infty$ widely employed in the theory of spin glasses. For this arrangement we solve a model Hamiltonian of the Josephson array in the thermodynamic limit ($N \to \infty$) and calculate the critical temperature for the glassy phase-locking transition, caused by the Josephson tunneling of bosons in random environment.

1. Introduction

Systems which exhibit the so-called glassy phases constitute the major attempt of the solid-state physics to address the issue of collective disorder. The importance of these systems results not only from the need to understanding particular materials. Rather, it is believed that the glassy ordering is of a qualitatively new kind, prototypical for a large class of random arrangements which go well beyond solid state physics. Systems which show spin-glass behavior have been intensively investigated theoretically, usually by employing massive numerical simulations [1]. Thus, it has proved extremely useful to consider the celebrated example of infinite-range Sherrington–Kirkpatrick (SK) model [2] as an archetypical system in spin glass theory, where an analytical approach was feasible. In the context of interacting bosons the influence of the *diagonal disorder* in quantum many-body systems is known giving rise to novel quantum glassy phases [3]. More recently, since the first experimental realizations of Bose–Einstein condensate (BEC) [4,5], quantum cold atomic gases have attracted physicists as ideal benchmark systems for testing of theoretical quantum statistical phenomena. Furthermore, atomic gases confined in magneto-optical traps have opened a new way for the study of strong correlated systems with unprecedented experimental control over physical parameters in these systems. For example, systems with strongly interactions can be set up using cold neutral atomic gases in optical lattices [6–8], which form periodic structure composed of micro-traps produced by standing wave laser beams. In contrast to the homogeneous trapped systems, optical lattices offer additional features regarding the degree of control over the physical parameters. Thus, physical properties of cold atoms can be studied e.g. as a function of a variety of physical parameters including: on-site inter-atomic interactions, tunneling amplitudes between adjacent sites, atom filling numbers as well as lattice dimensionality [9]. Furthermore, disorder can be generated in optical lattice systems by exposure to speckle lasers [10,11], incommensurate lattice-forming lasers [12–14], and by other methods [15]. Remarkably, while the effects of potential disorder have been widely investigated, other kinds of randomness, as for example the *off-diagonal* one, which affects hopping or interaction strengths, have remained unexplored.

By coupling of two BEC systems together, a weak link forms between them producing the so called Josephson junction [16]. This gives rise to a variety of phenomena as a result of the associated conjugate observables: number of bosons and their quantum-mechanical phase. For example, Josephson dynamics have been observed between weakly coupled macroscopic wave functions in Bose–Einstein condensates trapped in double well potentials [17–19]. Recent experiments have led to the creation of Joseph-

* Corresponding author. Fax: +48 71 321 4454.
E-mail addresses: kopec@ift.uni.wroc.pl, kopec@int.pan.wroc.pl (T.K. Kopeć).

http://dx.doi.org/10.1016/j.physleta.2016.11.021

Please cite this article in press as: T.K. Kopeć, M.M. Musiał, Sherrington–Kirkpatrick glassy-phase of random Josephson coupled Bose–Einstein condensates in wood-pile geometry, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2016.11.021

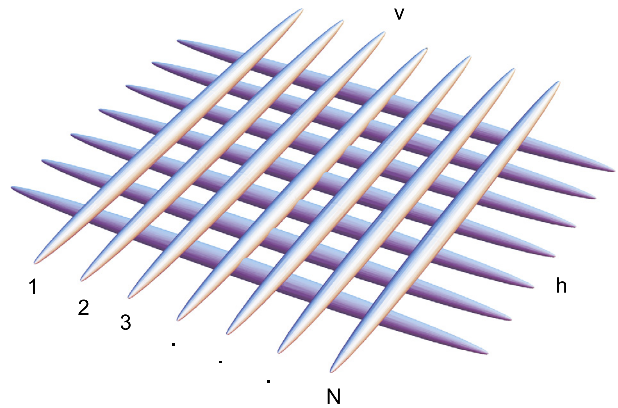

Fig. 1. Array of elongated Bose–Einstein condensates in wood-pile geometry: the arrangement of N vertical (v) and horizontal (h) elongated condensates mutually coupled via the random Josephson tunneling at each intersection.

son junction arrays based on the atomic BEC, where the atoms in a harmonic trap were additionally confined by an optical lattice potential, produced by far-detuned laser beams [20–23]. In the present work we will study a transition to a glassy phase of neutral atoms trapped in optical lattice, where the system is realized in a form of the array of individual Bose–Einstein condensates of N elongated vertical and horizontal rods in a wood-pile geometry mutually coupled via the random Josephson tunneling. In this geometry every horizontal (vertical) rod of a given condensate is linked to its vertical (horizontal) counterpart, so that the number of nearest neighbors z of a given rod in this system is equal to the number of condensate rods ($z = N$), implying that the system is fully connected. This feature together with superimposed randomness forms a prerequisite of the Sherrington–Kirkpatrick model for $N \to \infty$ widely known from the theory of spin glasses. For this arrangement we solve a model Hamiltonian of the Josephson array in the thermodynamic limit ($N \to \infty$) and calculate the critical temperature for the glassy phase-locking transition, caused by the Josephson tunneling of bosons in random environment. Our study of the disordered Josephson coupled condensates is motivated by the fact that these effects constitute experimentally testable signatures of the competition of disorder and phase coherence in superfluid systems [24].

2. The model with random Josephson couplings

For array of weakly coupled condensates with wood-pile geometry (see, Fig. 1), the Hamiltonian written in terms of the wave functions of the individual vertical (v) and horizontal (h) rods of Bose condensates takes the form [25]

$$\mathcal{H}(\{\mathcal{J}_{\ell\ell'}\}) = -\sum_{\ell=1}^{N}\sum_{\ell'=1}^{N}\mathcal{J}_{\ell\ell'}\left(\psi^{\star}_{v\ell}\psi_{h\ell'} + \psi^{\star}_{h\ell'}\psi_{v\ell}\right) + \epsilon\sum_{\ell=1}^{N}\left(|\psi_{v\ell}|^2 + |\psi_{h\ell}|^2\right) + \frac{U}{2}\sum_{\ell}\left(|\psi_{v\ell}|^4 + |\psi_{h\ell}|^4\right), \quad (1)$$

where $\psi_{\alpha\ell}$ is the complex wave function amplitude for the α-condensate in the $\alpha\ell$ rod ($\alpha = v, h$) located at the sites labeled by ℓ. Since each horizontal (vertical) condensate rod of the system is directly coupled to every other filament in the wood pile geometry the number of nearest neighbors z is $z = N$, where N stands for the number of rods in the upper (lower) plane of the system. The first term in the Hamiltonian in Eq. (1) contains the Josephson amplitude $\mathcal{J}_{\ell\ell'}$ and describes the tunneling of bosons between neighboring condensates, which we assume to be a random variable. As a result of the geometry involved the interactions $\mathcal{J}_{\ell\ell'}$ are infinite range in the limit $N \to \infty$, since the system is fully connected. To be specific we consider the Gaussian distribution with zero mean and the variance $J/\sqrt{N}$ given by

$$P(\{\mathcal{J}_{\ell\ell'}\}) = \sqrt{\frac{N}{2\pi J}}\exp\left(-\frac{N\mathcal{J}^2_{\ell\ell'}}{2J}\right). \quad (2)$$

Finally, the parameter U quantifies the on-site interaction energy, while ϵ describes the mean value of the trapping potential.

3. Disorder average and glassy phase order parameter

In the following we deal with the case where the disorder is assumed to be quenched. In this case that the variables $\mathcal{J}_{\ell\ell'}$ remain fixed while the U(1) phases of the wave functions of the individual condensates fluctuate. From an experimental point of view, this corresponds to a situation where the dynamical time scale of the disorder (e.g. the Josephson couplings between condensates) is much longer than the dynamical time scale of the phase fluctuations. Averages with respect to the probability distribution in Eq. (2) are defined through

$$[\ldots]_J = \int\prod_{\ell\ell'} d\mathcal{J}_{\ell\ell'} P(\mathcal{J}_{\ell\ell'})\ldots. \quad (3)$$

For example the distribution in Eq. (2), being Gaussian, is completely specified by its mean and standard deviation

$$[\mathcal{J}_{\ell\ell'}]_J = 0, \quad \left[\mathcal{J}^2_{\ell\ell'}\right]_J = J^2/N. \quad (4)$$

We would like to calculate the disorder-averaged free energy, which encodes all thermodynamic properties of this model

$$f_J \equiv [f(\{\mathcal{J}_{\ell\ell'}\})]_J = \int\prod_{\ell\ell'} d\mathcal{J}_{\ell\ell'} P(\mathcal{J}_{\ell\ell'}) f(\{\mathcal{J}_{\ell\ell'}\}). \quad (5)$$

The disorder dependent free-energy density is given by

$$f(\{\mathcal{J}_{\ell\ell'}\}) = \lim_{N\to\infty}\frac{F(\{\mathcal{J}_{\ell\ell'}\})}{N}, \qquad F(\{\mathcal{J}_{\ell\ell'}\}) = -\frac{1}{\beta}\ln Z(\{\mathcal{J}_{\ell\ell'}\}) \quad (6)$$

where $\beta = 1/k_B T$ with T being the temperature. Furthermore,

$$Z(\{\mathcal{J}_{\ell\ell'}\}) = \int\prod_{\ell=1}^{N} d^2\psi_{v\ell} d^2\psi_{h\ell} e^{-\beta H} \quad (7)$$

is the statistical sum, where $d^2\psi_{\alpha\ell} = d[\Re\mathrm{e}\psi_{\alpha\ell}]d[\Im\mathrm{m}\psi_{\alpha\ell}]$, ($\alpha = v, h$).

At certain finite temperature a new state eventually take place, which is marked by the competing effects of thermally driven phase fluctuations of individual condensates composing the system and phase locking due to random Josephson tunnel coupling with the freezing of U(1) phases of condensates into the disordered ground state. This new state is signaled by the non-zero value of the Edwards–Anderson glass order parameter defined by [1]

$$q_{\mathrm{EA}} = \left[\left\langle|\psi_{\alpha\ell}|^2\right\rangle_T\right]_J \quad (8)$$

where

$$\langle\ldots\rangle_T = \frac{1}{Z}\int\prod_{\ell=1}^{N} d^2\psi_{v\ell} d^2\psi_{h\ell}\ldots e^{-\beta H}. \quad (9)$$

As we already mentioned, because of the unique feature of the wood-pile geometry the number of nearest neighbors z in this

Please cite this article in press as: T.K. Kopeć, M.M. Musiał, Sherrington–Kirkpatrick glassy-phase of random Josephson coupled Bose–Einstein condensates in wood-pile geometry, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2016.11.021