Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Analytic description of Raman-induced frequency shift in the case of non-soliton ultrashort pulses

Aleksandr N. Bugay [a,*], Vyacheslav A. Khalyapin [b,c]

[a] Joint Institute for Nuclear Research, Joliot-Curie 6, 141980, Dubna, Moscow Region, Russia
[b] Immanuel Kant Baltic Federal University, Kaliningrad, 236041 Russia
[c] Kaliningrad State Technical University, Kaliningrad, 236000 Russia

ARTICLE INFO

Article history:
Received 14 July 2016
Received in revised form 7 October 2016
Accepted 10 November 2016
Available online xxxx
Communicated by V.A. Markel

Keywords:
Nonlinear optics
Raman effect
Intrapulse Raman scattering

ABSTRACT

Raman-induced frequency shift of ultrashort pulses have been studied extensively for the soliton propagation regime. Here we derive explicit analytic expressions for the evolution of Raman-induced frequency shift in much less studied case of non-soliton ultrashort pulses. Pulse spectra may belong to any region of group velocity dispersion including zero group dispersion point. The analysis is based on the moment method. Obtained expressions fit well to the numerical solution of the nonlinear wave equation.

1. Introduction

Raman-induced frequency shift (RIFS) or intrapulse Raman scattering (IRS) of ultrashort pulses propagating in optical fibers was discovered in 1985 [1]. In case of anomalous dispersion region this effect was observed in soliton propagation regime [2] and therefore it is usually referred as soliton self-frequency shift (SSFS) [3,4]. RIFS and SSFS have been observed in a number of various perspective photonic crystal fibers (PCF) [5–11]. Negative influence caused by these effects in high speed communication lines can be suppressed by special fiber design [12–16].

IRS effect, unlike classical Raman scattering, has no energy threshold, and the effect exists for any pulses, whose spectrum overlaps the band of vibrational medium resonances [17,18]. A wide range of intense non-soliton optical signals, whose profile is maintained by fiber dispersion, e.g. Gaussian and super-Gaussian pulses, Airy pulses, are subjected to action of IRS. Moreover, recently RIFS have been observed for dissipative structures in optical microresonator [19]. Therefore the problem of RIFS study in various optical systems is still important nowadays.

Theoretical study of IRS was started from the soliton case by the seminal work of Gordon [20], where the value of SSFS was shown to depend linearly on propagation distance along the fiber. However, a number of other effects acting at low timescales may interfere with Raman spectral shift resulting to its deceleration or saturation. It was shown that spectral shift is affected by pulse chirp [17], third-order dispersion [21–23], self-steepening of ultrashort light pulses [24], two-photon absorption [25], photoionization [7,11] and other effects.

The complexity of spectral dynamics of ultrashort pulses influenced by IRS is described theoretically by generalized nonlinear Schrödinger equation (GNLSE), which have to be solved numerically. Alternatively, approximate analytic methods provide much clear and fundamental understanding of the underlying phenomena. Such methods are usually based on the soliton perturbation theory [20,21,26,27] or variational methods such as the moment method [17,22,25,28–31] and the collective coordinate theory [32,33]. The latter two are more general in sense that they can be applied to study of non-soliton pulses.

The system of ordinary differential equations obtained for pulse parameters within the framework of discussed methods is easy for qualitative analysis. But the complete solution of this system is usually obtained by numerical computation, while treating the general method as 'analytic'. This is insufficient for complete understanding of phenomena and, especially, for prediction of new effects. Explicit analytic expressions were derived for the solitons [20–22,29], but not for the non-soliton pulses. Also, special case of pulses crossing zero group velocity dispersion point (ZDP) as a part of soliton spectral tunneling effect [26] was analyzed by numerical simulation only [29].

In this paper our main goal is to obtain analytic solution of equations obtained by the moment method and present much

* Corresponding author.
E-mail addresses: bugay_aleksandr@mail.ru (A.N. Bugay), slavasxi@gmail.com (V.A. Khalyapin).

http://dx.doi.org/10.1016/j.physleta.2016.11.018
0375-9601/

clear understanding of IRS phenomena. The dependencies of RIFS values for chirped non-soliton ultrashort pulses in the cases of both normal and anomalous group velocity dispersion will be derived in the form of the closed analytic expressions.

2. Theoretical model

The propagation of ultrashort pulses in a homogeneous dielectric (optical fiber) is described by a GNLSE of the form [4]

$$\frac{\partial\psi}{\partial z}+\frac{i\beta_2}{2}\frac{\partial^2\psi}{\partial\tau^2}-\frac{\beta_3}{6}\frac{\partial^3\psi}{\partial\tau^3}-i\gamma|\psi|^2\psi + \frac{\gamma}{\omega}\frac{\partial}{\partial\tau}\left(|\psi|^2\psi\right)+i\gamma T_R\psi\frac{\partial|\psi|^2}{\partial\tau}=0, \tag{1}$$

where ψ is the slowly varying envelope of the pulsed electric field

$$E=\frac{1}{2}\psi\exp[-i(\omega t-kz)]+c.c. \tag{2}$$

Here ω is the center frequency of the pulse spectrum, k is the wave-number, z is the propagation coordinate, $\tau=t-z/v_g$ is the retarded time, v_g is the group velocity taken at the frequency ω, β_2 is the group-velocity dispersion (GVD) coefficient, β_3 is the third-order dispersion parameter, γ is the nonlinearity coefficient and T_R is the Raman parameter.

The nonlinearity coefficient γ is usually positive for optical fibers. The coefficient of GVD β_2 is positive for normal GVD and negative for anomalous GVD. Third-order dispersion parameter β_3 is usually positive for typical telecom fibers like SMF-28.

We apply the moment method [17] mainly for the reason of its simplicity. Much more complicated collective coordinate theory may provide better fit to real dynamics, but the analytic treatment in this case is more difficult. The basic assumption for both methods is that the form of pulse envelope is considered a priori known at the desired propagation interval. The first situation is the quasi-linear propagation mode in dispersion-managed fibers, when a Gaussian-shaped pulse experiences dispersion spreading, but conserves its temporal profile [3]. Another situation is the soliton propagation mode [4], when a pulse maintains hyperbolic secant temporal profile.

The envelope of chirped Gaussian pulse can be written as [17]

$$\psi=A\exp\left[i\varphi-\frac{(1+iC)}{2}\left(\frac{\tau-T}{\tau_p}\right)^2+i\Omega(\tau-T)\right]. \tag{3}$$

Here A is the pulse amplitude, τ_p is the pulse temporal duration, C is the chirp parameter, T is the temporal delay, Ω is the red frequency shift, and the phase φ does not depend on τ. All the parameters depend on the propagation coordinate z. The initial values of chirp parameter, pulse duration and pulse amplitude are C_0, τ_0 and A_0, respectively. Frequency shift and temporal delay are assumed to be zero at the input $\Omega(0)=0$, $T(0)=0$.

According to the moment method the pulse parameters are found to evolve according to the following equation set [17,31]

$$\frac{d\tau_p}{dz}=(\beta_2-\beta_3\Omega)\frac{C}{\tau_p}, \tag{4}$$

$$\frac{dC}{dz}=(\beta_2-\beta_3\Omega)\left(\frac{1+C^2}{\tau_p^2}\right)+\frac{\gamma B}{\sqrt{2\pi}\,\tau_p}\left(1-\frac{\Omega}{\omega}\right), \tag{5}$$

$$\frac{dT}{dz}=-\beta_2\Omega+\frac{\beta_3}{2}\left(\Omega^2+\frac{1+C^2}{2\tau_p^2}\right)+\frac{3\gamma B}{2\sqrt{2\pi}\,\omega\tau_p}, \tag{6}$$

$$\frac{d\Omega}{dz}=\frac{\gamma B}{\sqrt{2\pi}\,\tau_p^3}\left(T_R-\frac{C}{\omega}\right), \tag{7}$$

$$B=\sqrt{\pi}A^2\tau_p=\text{const}. \tag{8}$$

Here we consider fibers with negligible linear losses, therefore the only source of dissipation is the absorption of pulse energy by molecular vibrations [34]. The constant quantity B in this case reflects the conservation of the photon number: the pulse spectrum experiences red shift, but the overall number of photons is conserved [24].

The value of frequency shift is rather small with respect to pulse carrier frequency $\Omega/\omega\ll 1$. In addition, we will consider pulses whose central frequency is far from the zero GVD point, i.e. $\Omega\beta_3/\beta_2\sim\Omega/\omega\ll 1$. Substitution of (4) in (7) and sequential integration result in general expression

$$\Omega=\frac{\gamma B}{\sqrt{2\pi}}\left[T_R\int_0^z\frac{dz}{\tau_p^3}-\frac{1}{\omega\beta_2}\left(\frac{1}{\tau_0}-\frac{1}{\tau_p}\right)\right]. \tag{9}$$

The first term here characterizes RIFS, and the second term characterizes the contribution of the self-steepening. As follows from (9) Raman effect produces red spectral shift, but the frequency shift produced by the self-steepening can be of variable sign.

The approximate general solution in case of dispersion spreading can be found from (4), (5) as a formal series expansion of τ_p^2 and C on z. Thus, we obtain the following expression for temporal duration

$$\left(\frac{\tau_p}{\tau_0}\right)^2=1+2gC_0\frac{z}{L_d}+\left(1+C_0^2+g\delta\right)\left(\frac{z}{L_d}\right)^2+\varepsilon z^2, \tag{10}$$

where $g=\text{sign}(\beta_2)$ denotes the sign of GVD, $L_d=\tau_0^2/|\beta_2|$ is the effective dispersion length. Dimensionless parameter $\delta=B/B_c\sim\gamma A^2$ characterizes relative influence of nonlinearity on pulse dynamics. Here $B_c=\sqrt{2\pi}|\beta_2|/(\gamma\tau_0)$ corresponds to soliton formation threshold. Also, the number N of solitons generated in case $\beta_2<0$ is defined by relation $N=\sqrt{\delta}$ [30]. Thus, $\delta=1$ corresponds to formation of fundamental soliton.

The last term in (10) takes into account high order effects, which cannot be extracted from (4). This correction can be found by the following procedure. One should calculate the second derivative $d^2(\tau_p^2)/dz^2$ using (1) and common definition [17] of squared temporal duration τ_p^2. Substitution of ansatz (3) gives new equation for τ_p, which refines (4). The remaining steps with series expansion are analogous to described above. Omitting cumbersome calculations we write the final result as

$$\varepsilon=\frac{\beta_3^2}{4\tau_0^6}\left[(1+C_0^2)^2-\frac{2\gamma B\tau_0}{\beta_3\sqrt{2\pi}}\left(T_RC_0-\frac{C_0^2-2}{2\omega}\right)\right]. \tag{11}$$

For pulses, whose frequency is far from ZDP this term can be safely neglected.

Notice that (10), (11) exactly coincides with known expression [3,35] for the pulse dispersion spreading in the linear propagation regime $B\to 0$. Thus, the expansion in (10) introduces nonlinear correction to the known exact formula and, therefore, the approximation is applicable for the weakly nonlinear propagation regime $\delta<1$.

Corresponding expression for the chirp parameter is

$$C=C_0+g(1+g\delta+C_0^2)\frac{z}{L_d}. \tag{12}$$

Substituting (10) in (9) we obtain

$$\frac{\Omega}{\omega}=\frac{\delta}{(\omega\tau_0)^2}\left[-g\left(1-\frac{\tau_0}{\tau_p(z)}\right)\left(1+\frac{\omega T_RC_0}{1+g\delta}\right)+\omega T_R\left(1+\frac{C_0^2}{1+g\delta}\right)\frac{\tau_0}{\tau_p(z)}\frac{z}{L_d}\right], \tag{13}$$