Results in Physics xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability

Aly R. Seadawy [a,b,*], Dianchen Lu [c,*]

[a] Mathematics Department, Faculty of Science, Taibah University, Al-Ula, Saudi Arabia
[b] Mathematics Department, Faculty of Science, Beni-Suef University, EgyptMathematics DepartmentFaculty of ScienceBeni-Suef UniversityEgypt
[c] Department of Mathematics, Faculty of Science, Jiangsu University, China

ARTICLE INFO

Article history:
Received 20 October 2016
Received in revised form 8 November 2016
Accepted 18 November 2016
Available online xxxx

MSC:
35G20
35Q53
37K10
49S05
76A60

Keywords:
Generalized higher order NLS equation
Solitary wave solutions
Mathematical Physics methods

ABSTRACT

The higher order nonlinear Schrödinger (NLS) equation describes ultra-short pluse propagation in optical fibres. By using the amplitude ansatz method, we derive the exact bright, dark and bright-dark solitary wave soliton solutions of the generalized higher order nonlinear NLS equation. These solutions for the generalized higher order nonlinear NLS equation are obtained precisely and efficiency of the method can be demonstrated. The stability of these solutions and the movement role of the waves are analyzed by applying the modulation instability analysis and stability analysis solutions. All solutions are exact and stable.

1. Introduction

Nonlinear wave phenomena exist in many fields, such as fluid mechanics, plasma physics, biology, hydrodynamics, solid state physics and optical fibers, etc. In order to better understand these nonlinear phenomena, it is important to seek their exact solutions. They can help to analyze the stability of these solutions and the movement role of the wave by making the graphs of the exact solutions [1–11].

Optical solitons have been the subjects of extensive research in nonlinear optics due to their potential applications in telecommunication and ultra fast signal processing systems [12–14]. Optical solitons arise from the balance between group velocity dispersion effect and nonlinear effect arising due to nonlinear change in the refractive index [13]. A higher-order Schrodinger equation containing parameters, which is used to describe pulse propagation in optical fibres, is shown to admit an infinite-dimensional prolongation structure for exactly four combinations of the parameters, besides the classical NLS equation [15–23].

Within the framework of the Madelung fluid description, the bright and dark (including gray- and black-soliton) envelope solutions for a generalized mixed NLS equation were deduced, by virtue of the corresponding solitary wave solutions for the generalized stationary Gardner equations [24]. The connection between the envelope soliton-like solutions of a wide family of NLS equations and the soliton-like solutions of a wide family of KdV equations was derived. Under suitable hypothesis for the current velocity, the Gerdjikov–Ivanov envelope solitons were analyzed and deduced [25–27] (Fig. 1).

Consider one of the integrable cases of the generalized higher order nonlinear Schrödinger (NLS) equation [12,13]

$$i\frac{\partial q}{\partial x}+\alpha_1\frac{\partial^2 q}{\partial t^2}+\alpha_2|q|^2q+i\epsilon\alpha_3\frac{\partial^3 q}{\partial t^3}+i\epsilon\alpha_4|q|^2\frac{\partial q}{\partial t}+i\epsilon\alpha_5 q\frac{\partial|q|^2}{\partial t}=0, \qquad (1)$$

where x and t are the normalized distance of the propagation and retarded time; $q(x,t)$ is the slowly varying envelope of the electric field; ϵ denotes the ratio of the width of the spectra to the carrier

* Corresponding authors at: Mathematics Department, Faculty of Science, Taibah University, Al-Ula, Saudi Arabia (A.R. Seadawy).
E-mail addresses: Aly742001@yahoo.com (A.R. Seadawy), dclu@ujs.edu.cn (D. Lu).

http://dx.doi.org/10.1016/j.rinp.2016.11.038
2211-3797/© 2016 The Author. Published by Elsevier B.V.

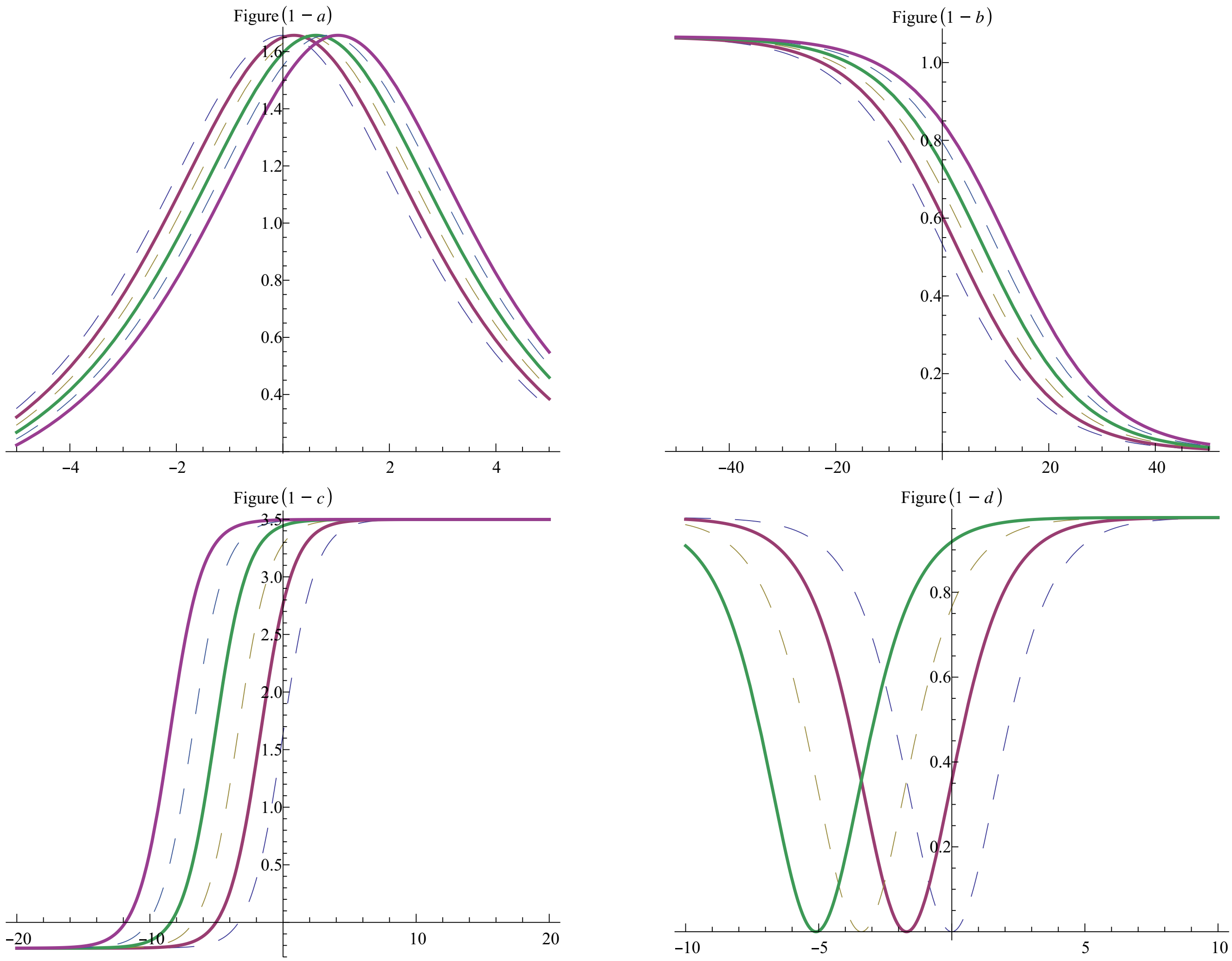

Fig. 1. Exact soliton solutions in one dimension for generalized NLS equation, shapes are plotted (bright and dark solitary wave solutions): (1a)–(1d); One-dimensional soliton solutions in the different intervals.

frequency; $\alpha_1, \alpha_2, \alpha_3$, α_4 and α_5 are real parameters that related the the effects of the group velocity dispersion, self-phase modulation, third-order dispersion, self-steepening and stimulated Raman scattering. In order to solve the generalized higher order NLS equation, we suppose

$$q(x,t) = u(x,t)e^{i\phi(x,t)}, \quad q^*(x,t) = u(x,t)e^{-i\phi(x,t)}, \quad \phi(x,t) = kx - \omega t, \tag{2}$$

where * denotes a complex conjugate, $\phi(x,t)$ is the linear phase shift function with k and ω are the normalized wave vector and frequency. By substituting from the solution (2) in the generalized higher order NLS Eq. (1), we obtain

$$(-k + \alpha_1\omega^2 - \epsilon\alpha_3\omega^3)u + (\alpha_2 + \epsilon\alpha_4\omega)u^3 + i\frac{\partial u}{\partial x} + (3i\epsilon\alpha_3\omega^2 - 2i\alpha_1\omega)\frac{\partial u}{\partial t} + (i\epsilon\alpha_4 + 2\epsilon\alpha_5\omega)u^2\frac{\partial u}{\partial t} + (\alpha_1 + 3\epsilon\alpha_3\omega^2)\frac{\partial^2 u}{\partial t^2} + i\epsilon\alpha_3\frac{\partial^3 u}{\partial t^3} = 0, \tag{3}$$

1.1. Stability analysis solutions

Hamiltonian system is a mathematical formalism to describe the evolution equations of a physical system. By using the form of a Hamiltonian system for which the momentum is given as

$$M = \lim_{s\to\infty}\int_0^s |u|_i^2 dx, \tag{4}$$

where $i = 1,2,3,4,5$. The sufficient condition for solitary wave solutions stability is

$$\frac{\partial M}{\partial \omega} > 0. \tag{5}$$

where ω is the frequency. By using Eqs. (4) and (5), we discuss the criteria for stability of solutions for the generalized higher order NLS equation in given cases (Fig. 2).

2. Solitary wave solutions

Case I: We take the ansatz function of the generalized higher order NLS equation in the form of bright solitary wave solution:

$$u_1(x,t) = A\mathrm{sech}\left(w\left(t - \frac{x}{v}\right)\right), \quad q(x,t) = A\mathrm{sech}\left(w\left(t - \frac{x}{v}\right)\right)e^{i(kx-\omega t)}, \tag{6}$$

where A, w, v are the amplitude, the pulse width and velocity of soliton in normalized unites. Substituting from Eq. (6) in Eq. (1), and separating the real and imaginary parts, we obtain

Please cite this article in press as: Seadawy AR, Lu D. Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability. Results Phys (2016), http://dx.doi.org/10.1016/j.rinp.2016.11.038