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Radiative flow due to stretchable rotating disk with variable thickness

Tasawar Hayat [a,b], Sumaira Qayyum [a,*], Maria Imtiaz [a], Ahmed Alsaedi [b]

[a] Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan
[b] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

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ABSTRACT

Present article concerns with MHD flow of viscous fluid by a rotating disk with variable thickness. Heat transfer is examined in the presence of thermal radiation. Boundary layer approximation is applied to the partial differential equations. Governing equations are then transformed into ordinary differential equations by utilizing Von Karman transformations. Impact of physical parameters on velocity, temperature, skin friction coefficient and Nusselt number is presented and examined. It is observed that with an increase in disk thickness and stretching parameter the radial and axial velocities are enhanced. Prandtl number and radiation parameter have opposite behavior for temperature field. Skin friction decays for larger disk thickness index. Magnitude of Nusselt number enhances for larger Prandtl number.

Introduction

Presently the flow by rotating surfaces is very popular area of research. It is because of its relevance in engineering and industrial sectors including jet motors, food processing, electric power generating system and turbine system. Theoretical and experimental studies for this type of flow seem interesting. Pioneer work on flow due to rotating disk is done by Karman [1]. He provided transformations which help us to construct ordinary differential equation from Navier Stokes equations. Cochran [2] also used these transformations to examine rotating disk flow by numerical integration method. Rotating flow by two disks is firstly examined by Stewartson [3]. Chapple and Stokes [4] and Mellor et al. [5] studied flow between rotating disks. Heat transfer between two rotating disks is explored by Arora and Stokes [6]. Kumar et al. [7] described flow phenomenon between porous stationary disk and solid rotating disk. Hayat et al. [8] analyzed thermal stratification effects in rotating flow between two disks. Radiative flow of carbon nanotubes between rotating stretchable disks with convective conditions is studied by Hayat et al. [9]. Anderson et al. [10] and Ming et al. [11] examined the flow and heat transfer of power law fluid by a rotating disk for unsteady and steady cases respectively.

Surfaces of variable thickness have applications in engineering particularly mechanical, architectural, civil, marine and aeronautical processes. It also helps to reduce the weight of structural elements and improve the utilization of material. However it is noted that very little literature is present for flow due to stretching surfaces having variable thickness. Hayat et al. [12] examined flow over variable thicked surface with variable thermal conductivity and Cattaneo-Christov heat flux. Ramesh et al. [13] studied Casson fluid flow over variable stretching sheet with thermal radiation. Xun et al. [14] analyzed Ostwald-de Waele fluid flow by rotating disk of variable thickness with index decreasing. Hayat et al. [15] worked on stagnation point flow past a variable thicked surface with nonlinear stretching and carbon nanotube effects. Fang et al. [16] described the flow over stretching sheet with variable thickness. Flow of Williomson nanofluid over a stretching sheet with variable thickness has been examined by Hayat et al. [17]. Zhang et al. [18] worked on bending collapse of square tubes with variable thickness. Hayat et al. [19] discussed the homogeneous-heterogeneous reactions and melting heat transfer effects in the flow by a stretching surface with variable thickness. Acharya et al. [20] analyzed the variable thickness on nanofluid flow by a slendering stretching sheet. Khader and Megahed [21] examined the partial slip effects on boundary layer flow due to a nonlinearly stretching sheet with variable thickness.

Radiation has many applications in engineering as well as industrial sector such as nuclear reactor, glass production, furnace design, power plant and also in space technology. In radiation process the electromagnetic waves are responsible for transfer of energy which carry energy away from the emitting object. Radiative flow and heat transfer over a permeable stretching sheet with Hall current are analyzed by Pal [22]. Hayat et al. [23] studied stretched flow of nanofluid in presence of nonlinear thermal radiation and mixed convection. Sheikholeslami et al. [24] worked on

* Corresponding author.
E-mail address: sarak0674@gmail.com (S. Qayyum).

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radiative flow of nanofluid. Flow due to porous wedge in presence of mixed convection and thermal radiation are examined by Rashidi et al. [25]. Hayat et al. [26] studied partial slip effects in radiative flow of nanofluid. MHD flow of Oldroyd-B nanofluid with radiative surface is analyzed by Shehzad et al. [27]. Battacharyya et al. [28] described radiative flow of micropolar fluid over a porous shrinking sheet.

There are many methods to solve the nonlinear problems. Homotopy analysis method (HAM) is firstly developed by S. Liao in 1992 [29]. He further modified [30] with a non-zero auxiliary parameter which is also known as convergence control parameter $\hbar$. This parameter is a non-physical variable that provides a simple way to verify and ensure convergence of solution series. The HAM always helps no matter whether there exist small/large physical parameters or not in the problem statement. It provides a convenient way to guarantee the convergence of approximation series. It also provides great freedom to choose the equation type of linear sub-problems and the base functions of solutions.The capability of the HAM to naturally show convergence of the series solution is unusual in analytical and semi-analytic approaches to nonlinear partial differential equations. There are also some disadvantages that mathematical foundation of this method is not very well established and it will not work for cases with zero radius of convergence. The method has been used by several authors and proved to be very effective in deriving an analytic solution especially for nonlinear differential equation [31–42].

Much attention in past has been given to the flow due to rotating disk with negligible thickness. It is hard to find any such study in presence of radiation. Furthermore the radiative flow by stretchable rotating disk having variable thickness is not studied yet. Our main objective is to fill this void. MHD effects are also taken in account. Graphical technique is used to elaborate the impact of involved parameters on velocity, temperature, skin friction coefficient and Nusselt number.

Modeling

We consider steady incompressible flow due to a stretchable rotating disk with angular velocity Ω and stretching rate c. Fluid occupies the semi infinite region over the disk with variable thickness and surface is considered at $z = a\left(\frac{r}{R_0}+1\right)^{-\varsigma}$. Temperature at the surface of the disk is $\widehat{T}_w$ and ambient temperature is assumed $\widehat{T}_\infty$. Magnetic field of strength B_0 is applied parallel to z-axis. We are considering cylindrical coordinates (r, θ, z) and physical model is presented in Fig. 1. Under the assumptions

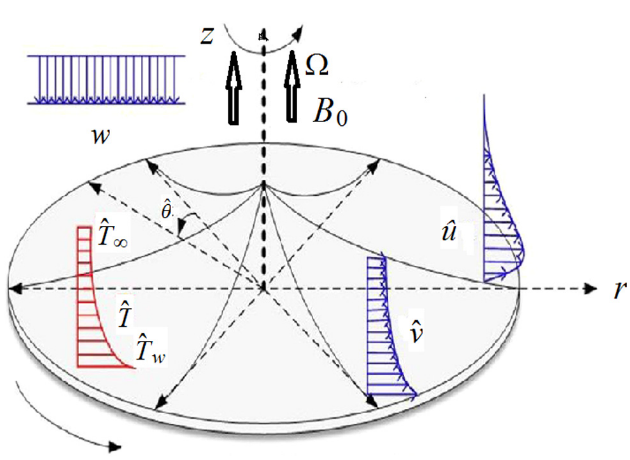

Fig. 1. Flow geometry.

$\frac{\partial \hat{p}}{\partial r} = \frac{\partial \hat{p}}{\partial z} = 0, O(\hat{u}) = O(\hat{v}) = O(r) = O(1)$ and $O(\hat{w}) = O(z) = O(\delta)$ the equations for flow and heat transfer [14] are as follows:

$$\frac{\partial \hat{u}}{\partial r} + \frac{\hat{u}}{r} + \frac{\partial \hat{w}}{\partial z} = 0, \tag{1}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial r} + \hat{w}\frac{\partial \hat{u}}{\partial z} - \frac{\hat{v}^2}{r} = \nu\frac{\partial^2 \hat{u}}{\partial z^2} - \frac{\sigma}{\rho}B_0^2\hat{u}, \tag{2}$$

$$\hat{u}\frac{\partial \hat{v}}{\partial r} + \hat{w}\frac{\partial \hat{v}}{\partial z} + \frac{\hat{u}\hat{v}}{r} = \nu\frac{\partial^2 \hat{v}}{\partial r^2} - \frac{\sigma}{\rho}B_0^2\hat{v}, \tag{3}$$

$$(\rho c_p)\left(\hat{u}\frac{\partial \widehat{T}}{\partial r} + \hat{w}\frac{\partial \widehat{T}}{\partial z}\right) = \left(k + \frac{16\sigma^\circ T_\infty^3}{3k^\circ}\right)\frac{\partial^2 \widehat{T}}{\partial z^2}, \tag{4}$$

with boundary conditions

$$\hat{u} = rc, \quad \hat{v} = r\Omega, \quad \hat{w} = 0, \quad \widehat{T} = \widehat{T}_w \text{ at } z = a\left(\frac{r}{R_0}+1\right)^{-\varsigma},$$
$$\hat{u} = 0, \quad \hat{v} = 0, \quad \hat{w} = 0, \quad \widehat{T} = \widehat{T}_\infty \text{ at } z \to \infty, \tag{5}$$

where ν denotes kinematic viscosity, σ the electrical conductivity, ρ the density, c_p the specific heat, k the thermal conductivity, σ° the stefan–Boltzmann constant, k° the mean absorption coefficient, a is the thickness coefficient of the disk which is very small, R_0 the feature radius and ς the disk thickness index. Generalized Von Karman transformations are

$$\hat{u} = r^*R_0\Omega\widetilde{F}(\eta), \quad \hat{v} = r^*R_0\Omega\widetilde{G}(\eta), \quad \hat{w} = R_0\Omega(1+r^*)^{-\varsigma}\left(\frac{\Omega R_0^2\rho}{\mu}\right)^{\frac{-1}{n+1}}\widetilde{H}(\eta)$$

$$\tilde{\vartheta} = \frac{\widehat{T} - \widehat{T}_\infty}{\widehat{T}_w - \widehat{T}_\infty}, \quad \eta = \frac{z}{R_0}(1+r^*)^{\varsigma}\left(\frac{\Omega R_0^2\rho}{\mu}\right)^{\frac{1}{n+1}}. \tag{6}$$

After using transformations Eqs. (1)–(5) take the form

$$2\widetilde{F} + \widetilde{H}' + \eta\epsilon\varsigma\widetilde{F}' = 0, \tag{7}$$

$$\widetilde{F}''(\mathrm{Re})^{\frac{1-n}{1+n}}(1+r^*)^{2\varsigma} - \widetilde{F}^2 + \widetilde{G}^2 - \widetilde{H}\widetilde{F}' - \widetilde{F}\widetilde{F}'\varsigma\eta\epsilon - M\widetilde{F} = 0, \tag{8}$$

$$\widetilde{G}''(\mathrm{Re})^{\frac{1-n}{1+n}}(1+r^*)^{2\varsigma} - 2\widetilde{F}\widetilde{G} - \widetilde{H}\widetilde{G}' - \widetilde{F}\widetilde{G}'\varsigma\eta\epsilon - M\widetilde{G} = 0, \tag{9}$$

$$\frac{1}{\mathrm{Pr}}(1+R)(\mathrm{Re})^{\frac{1-n}{1+n}}(1+r^*)^{2\varsigma}\tilde{\vartheta}'' - \widetilde{F}\tilde{\vartheta}'\varsigma\eta\epsilon - \widetilde{H}\tilde{\vartheta}' = 0, \tag{10}$$

with boundary conditions

$$\widetilde{H}(\alpha) = 0, \quad \widetilde{F}(\alpha) = A, \quad \widetilde{F}(\infty) = 0, \quad \widetilde{G}(\alpha) = 1,$$
$$\widetilde{G}(\infty) = 0, \quad \tilde{\vartheta}(\alpha) = 1, \quad \tilde{\vartheta}(\infty) = 0, \tag{11}$$

where

$$\mathrm{Re} = \frac{\Omega R_0^2}{\nu}, \quad \mathrm{Pr} = \frac{\rho c_p \nu}{k}, \quad \epsilon = \frac{r^*}{R_0 + r^*}, \quad A = \frac{c}{\Omega},$$
$$R = \frac{16\sigma^\circ T_\infty^3}{3kk^\circ}, \quad \alpha = \frac{a}{R_0^2}\left(\frac{\Omega R_0^2\rho}{\mu}\right)^{\frac{-1}{n+1}} \tag{12}$$

where ϵ is a dimensionless constant, ς is disk thickness index, Re denotes Reynolds number, Pr is Prandtl number, A is scaled stretching parameters, R is radiation parameter, r^* is the dimensionless radius and M is magnetic parameter.

We now introduce deformations

$$\widetilde{H} = \tilde{h}(\eta - \alpha) = \tilde{h}(\xi), \quad \widetilde{F} = \tilde{f}(\eta - \alpha) = \tilde{f}(\xi),$$
$$\widetilde{G} = \tilde{g}(\eta - \alpha) = \tilde{g}(\xi), \quad \tilde{\vartheta} = \tilde{\theta}(\eta - \alpha) = \tilde{\theta}(\xi). \tag{13}$$

Eqs. (7)–(11) are reduced to the forms: