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Soret and Dufour effects on Jeffery-Hamel flow of second-grade fluid between convergent/divergent channel with stretchable walls

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ABSTRACT

This article investigates the problem related to Jeffery-Hamel with stretchable walls. The effects of mass and heat transfer are taken into account. Soret, Dufour and viscous dissipation effects are examined to scrutinize the behavior of concentration and temperature profiles between convergent/divergent stretchable channels. By using similarity variables, nonlinear partial differential equations governing the flow are reduced into the nondimensional coupled system of ordinary differential equations. Then, said flow model is tackled analytically and numerically over a prescribed domain. For analytical solution, we employed Homotopy Analysis method while for numerical once, Runge-Kutta scheme is used after reducing the system of ordinary differential equations into system of first order initial value problem. The effects of various dimensionless parameters that ingrained in the velocity, concentration and temperature fields are scrutinized graphically for convergent and divergent stretchable channels. Also, the values of skin friction coefficient, local Nusselt and Sherwood numbers are calculated analytically by using Homotopy Analysis method (HAM) for different physical parameters. Different values of quantities of physical interest (skin friction coefficient, local rate of heat and mass transfer) are tabulated for different ingrained parameters in the flow model. Finally, comparison between present results with already existing results in the literature has been made.

Introduction

Jeffery in 1915 [1] and Hamel [2] in 1916 studied a type of flow which is named after them as Jeffery-Hamel flow. It is two dimensional flow bounded between two nonparallel walls. Flow is caused by a source or sink at the intersection of the channel walls. Scrutinizes of Jeffery-Hamel flow, mass and heat transfer of second-grade fluid between convergent and divergent channels are very significant motive. This type of flow has numerous uses in the field of chemical, mechanical, aerospace, environmental and civil engineering.

Due to wide range of applications in industries and engineering, inspection of non-Newtonian fluids has been very significant motive. These type of fluids cannot be investigated by using Newton's law of viscosity and Navier-Stokes equations are unproductive for their flow impersonation. The non-Newtonian fluids cannot be investigated by exploiting a single constitutive relation. Hence for the non-Newtonian fluids, several constitutive equations are intended. Systematization of these fluids fall into three categories namely integral, differential and rate. Also, the governing equations for such type of fluids are highly nonlinear and more tedious. The issue of nonlinearity and boundary conditions of the involved equations limit the solutions of these fluids. The model of second-grade fluid has much attention for certain subclasses of differential type fluids.

In 2009 C. Wu et al. [3] explored the problem for second-grade fluid. They calculated the numerical solution of Stokes first problem for a heat generalized second-grade fluid with fractional derivative. In 2011, B.I. Olajuwon [4] studied the hydromagnetic flow of second-grade fluid with thermal diffusion and thermal radiation. Also, they discussed convection heat and mass transfer. In 2012 M. Jamil et al [5] explored the analytical solutions for helical flows of second grade fluids. M. Esmaeilpour [6] studied the problem of Jeffery-Hamel flow. For analytical solution of the problem, they used Optimal Homotopy asymptotic method. Flow of viscoelastic fluids between converging/diverging channels with magnetohydrodynamic effects are inspected by K. Sadeghy [7] in 2007. In 2010 Y. Yao et al. [8] investigated the problem of second grade fluid and discussed the unsteady flows over a plane wall. S.

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S. Motsa et al. [9] discussed the MHD Jeffery-Hamel flow problem. For solution of the problem, they employed spectral Homotopy Analysis method.

Recently, A. Gul et al. [10,11] studied the flow model of mixed convection flow along a vertical channel and energy transfer in mixed convection flow of nanofluid containing different shapes of nanoparticles. They also discussed the magnetohydrodynamic effects on the flow. Radiation and heat generation effects in magnetohydrodynamic mixed convection flow of nanofluid was reported by A. Gul et al. [12]. In 2016, N. Athirah et al. [13] explored the problem of nanoparticles free convection flow of a Jeffery fluid over an oscillating vertical plate. The influence of MHD in the flow field is also the part of his discussion. A. Khalid et al. [14] investigated the heat transfer analysis of ferrofluid with cylindrical shape nanoparticles past a vertical plate with ramped wall temperature embedded in a porous medium. MHD flow of water based Brinkman type nanofluid over a vertical plate was inspected by F. Ali et al. [15]. S. Aman et al. [16] studied the influence of gold nanoparticles on MHD mixed convection Poiseuille flow of nanofluid passing through a porous medium in the presence of thermal radiation, thermal diffusion and chemical reaction.

M. Sheikholeslami [17] explored CVFEM for magnetic nanofluid convective heat transfer in a porous curved enclosure. They also [18] discussed the impacts of Lorentz forces on nanofluid flow in a porous cylinder considering Darcy model. M. Sheikholeslami et al. [19] also reported two phase nanofluid model analysis in existence of induced magnetic field. Influence of Coulomb forces on $Fe_3O_4 - H_2O$ nanofluid thermal improvement was explored by M. Sheikholeslami [20] in 2016. Numerical treatment of external magnetic source impact on water based nanofluid convective heat transfer investigated by M. Sheikholeslami et al. [21] in 2016. In 2014, M.S. Kandelousi, discussed the impact of spatially variable magnetic field on ferrofluid flow and heat transfer considering constant heat flux boundary condition [22]. KKL correlation for simulation of nanofluid flow and heat transfer in a permeable channel explored by M. Sheikholeslami et al. [23] in 2014. Influence of uniform suction on nanofluid flow in cylinder studied by [24] in 2014. Flow of ferro-nanofluid in cavity with wavy wall was reported by [25].

Exact solutions are inconceivable for nonlinear differential equations. Many authors calculated the series solutions for such problems by using analytical methods like Adomian's Decomposition method (ADM) [26–28], Variation of Parameters (VPM) [29–31], Homotopy Analysis method (HAM) [32–34], Differential Transform method (DTM) [35,36] and Homotopy Perturbation method (HPM) [37,38]. Also, we can study [39], [40], [41] and [42].

From literature survey, it is revealed that, for second grade fluid no attempt has been made to study the Dufour and Soret effects on second-grade Jeffery-Hamel flow in converging and diverging channels with stretchable walls. Exact solutions are not credible for highly nonlinear differential equations. Thus, we calculated the series solution of the problem. For this purpose, we employed semi analytical method known as Homotopy Analysis method (HAM). The variations in velocity, concentration and temperature profiles for varying physical parameters embedded in the flow problem are discussed graphically. Furthermore, results for interest of physical quantities, Skin friction coefficient, local Nusselt and local Sherwood numbers are calculated analytically by using HAM. Finally, some concluding remarks regarding to this work.

Description of the problem

Contemplated a two dimensional steady flow of second grade fluid between two nonparallel walls having a source at the cusp of the walls. Flow is assumed to be purely radial and unidirectional. Thus only u component of the velocity survive and it is a function of both r and θ. Thus, $\boldsymbol{V} = (u, 0, 0)$. Furthermore, flow is assumed to be symmetric in nature. Fig. 1 represents the geometry of the flow model.

Generally, the continuity, momentum, energy and concentration equations are;

$$\nabla.\boldsymbol{V} = \boldsymbol{0} \tag{1}$$

$$\rho\frac{d\boldsymbol{V}}{dt} = \nabla.\boldsymbol{R}, \tag{2}$$

$$\rho\frac{de}{dt} = \boldsymbol{R}.\boldsymbol{W} - \nabla.\mathbf{q} \tag{3}$$

$$\boldsymbol{V}.\nabla C^* = \nabla.\left(D\nabla C^* + \frac{D_T}{T_m}\nabla T\right), \tag{4}$$

$$\boldsymbol{R} = -p\boldsymbol{I} + \tau, \tag{5}$$

$$\tau = \mu\boldsymbol{T}_1 + \alpha_1\boldsymbol{T}_2 + \alpha_2\boldsymbol{T}_1^2 \tag{6}$$

$$\boldsymbol{T}_1 = \boldsymbol{W} + \boldsymbol{W}^*, \tag{7}$$

$$\boldsymbol{W} = \nabla\boldsymbol{V}, \tag{8}$$

$$\boldsymbol{T}_2 = \frac{d\boldsymbol{V}}{dt} + \boldsymbol{T}_1\boldsymbol{W} + \boldsymbol{W}^*\boldsymbol{T}_1, \tag{9}$$

In Eqs. (1)–(8), pressure, identity tensor, shear stress, first and second Rivlin Ericksin tensors, laplacian and material constants are denoted by $p, I, \tau, \boldsymbol{T_1}, \boldsymbol{T_2}, \nabla, \alpha_1, and \alpha_2$, respectively. On solving Eqs. (1)–(8), we have the following momentum, temperature and concentration equations in dimensional form. Also, similarity variables and supporting boundary conditions are given by Eqs. (14) and (15), respectively [43,44].

$$\begin{aligned}\frac{u\partial u}{\partial r} = & -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(2\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{2u}{r^2}\right) \\ & -\frac{\alpha_1}{\rho}\left(\frac{2u}{r}\frac{\partial^2 u}{\partial r^2} + 2\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r^2} - \frac{2}{r^3}\left(\frac{\partial u}{\partial \theta}\right)^2 + 2u\frac{\partial^3 u}{\partial r^3} - \frac{1}{r^2}\frac{\partial u}{\partial \theta}\frac{\partial^2 u}{\partial r\partial \theta}\right) \\ & -\frac{\alpha_1}{\rho}\left(\frac{1}{r^2}\frac{\partial^2 u}{\partial r\partial \theta} + \frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial \theta^2} - \frac{2u}{r^3}\frac{\partial^2 u}{\partial \theta^2} + \frac{u}{r^2}\frac{\partial^3 u}{\partial r^2\partial \theta} + 2\frac{u^2}{r^3} - \frac{2u}{r^2}\frac{\partial u}{\partial r}\right)\end{aligned} \tag{10}$$

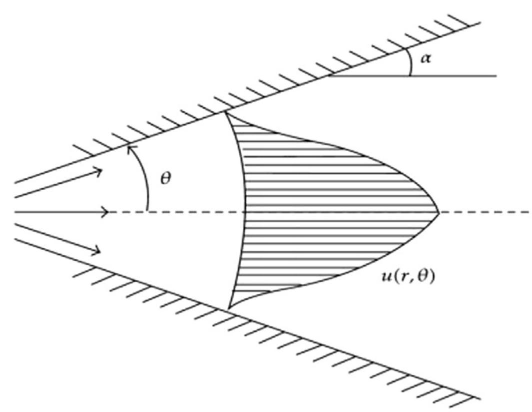

Fig. 1. Geometry of the flow model.