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Skeleton extraction based on the topology and Snakes model

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ABSTRACT

A new skeleton line extraction method based on topology and flux is proposed by analyzing the distribution characteristics of the gradient vector field in the Snakes model. The distribution characteristics of the skeleton line are accurately obtained by calculating the eigenvalues of the critical points and the flux of the gradient vector field. Then the skeleton lines can be effectively extracted. The results also show that there is no need for the pretreatment or binarization of the target image. The skeleton lines of complex gray images such as optical interference patterns can be effectively extracted by using this method. Compared to traditional methods, this method has many advantages, such as high extraction accuracy and fast processing speed.

Introduction

Skeleton is considered to be one of the most effective methods to analyze and identify the object shape, because it can concisely describe the basic shape of the object, and reduce the amount of information required to describe the object. And it is widely used in the analysis of optical interference measurement results [1–3]. Optical interference measurement is a very important method in the photoelastical mechanics, and it has advantages of high sensitivity, high accuracy, high precision and is easy for field application. The light source we obtained during the process of light interference is non-ideal monochromatic light, which would be subject to all kinds of outside interference. Thus the intensity distribution of the light is not uniform, and the spatial distribution of interference patterns varies in a large scope. Meanwhile, the maximal or minimal of optical interference image gray value is usually not in the geometric center. Therefore, it is unable to obtain relatively accurate extraction results especially for the opticalelastic interference pattern with large strip width or electronic speckle interference pattern with loud noise with conventional methods such as Blum's model [4], minimal spanning tree [5], the medial axial transformation model [6]. Obviously, these skeleton extraction methods appeared to be inadequate.

Optical interference pattern as an important information carrier plays a vital role in the accuracy and timeliness of the information extraction as well as the analyze of the measurement result. So to propose an accurate skeleton extraction algorithm has practical significance in improving the accuracy of Optical Interferometry Measurement and its development. In this paper, a new method of skeleton line extraction is proposed, which can accurately extract the skeleton lines of the optical interference gray image. The gradient vector field of the target image is obtained firstly with this method. Then saddle point and the node are to be detected. At the same time, new physical momentum flux is introduced to analyze the flux magnitude in gradient vector field. Finally, the skeleton is obtained based on the detection results of the saddle point in gradient vector flow field and the node as well as flux magnitude.

Gradient vector filed

The static vector field in the classical Snakes model could be decomposed into a gradient vector field with the non-rotational and non-dispersive components simultaneously according to the Helmholtz theorem [7]. The innovation of GVF Snakes model is to process the image force of the classical Snakes model by using the diffusion equation and get the gradient vector field of the whole image domain as an external force. The GVF is more orderly than the image force and present the object boundary of macroscopic tendency more clearly after the diffusion equation processing. At the same time, the gradient vector flow is obtained through the image field edge conduction instead of converting the energy function into a negative gradient. GVF Snakes model is a process to optimize utilizing force equilibrium conditions, because the GVF is not an expression, which cannot be solved by energy function.

Make $f(x,y)$ as the image force, then the energy function of the Snakes GVF model can be expressed as [7]:

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$$E = \int\int \mu\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right) + |\nabla f|^2 |V - \nabla f|^2 dxdy \tag{1}$$

Here, $V(x,y) = [u(s), v(s)] = [u(x,y), v(x,y)]$, u_x, u_y and v_x, v_y are partial derivative of $u(x,y)$ and $v(x,y)$ respectively. Above formula shows that, when ∇f is very small, the energy function is mainly affected by the square and the partial derivative; while ∇f is large, it is mainly affected by the second term; and when $V(x,y) = \nabla f$, the entire energy function tend to obtain the minimum value.

In the process of solving the energy function, the solution of formula (1) can be obtained by solving the following Euler equation, namely the Euler equations the energy function of the Snakes GVF model corresponds to are:

$$\begin{cases} \mu\nabla^2 u - (u - f_x)\left(f_x^2 + f_y^2\right) = 0 \\ \mu\nabla^2 v - (v - f_x)\left(f_x^2 + f_y^2\right) = 0 \end{cases} \tag{2}$$

Here, ∇^2 is Laplace operator. Formula (2) is considered to be a generalized diffusion equation, which expands ∇f outward. When $|\nabla f|$ is very small, the first term in the formula (2) plays a leading role (smoothing term), which produces a slowly varying field; When $|\nabla f|$ is large, the second term plays a leading role (gradient term); When $v = \nabla f$, the energy function E tends to reach the minimum value. And formula (2) also shows that, when the $\mu = 0$, formula (2) degenerates to a classic image force field, while u increases smoothing effect also increases, making the GVF scope expand correspondingly. Smoothing term also eliminates the noise influence to some extent.

Fig. 1 is the H model image for testing, and the H model GVF obtained by theoretical calculation. The figure shows that each point in the GVF is a vector, and each vector is different in magnitude and direction, so each vector can be used to represent the local characteristics of the image. Meanwhile, Fig. 1(b) shows that there are a large number of vectors convergence in image, whose location fits perfectly the skeleton location. In addition, Fig. 1 also shows that the image features more pronounced with GVF dealing with the underlying image. In Fig. 1(b), the skeleton location is quite obvious. To obtain accurate GVF of the target image is a prerequisite for accurate skeleton extraction. Meanwhile, to design a set of reasonable algorithm, making geometric problems digital and extracting obvious characteristics of image feature point, and then to do further optimization, the skeleton position of the target image can be determined.

Topology analysis and flux

As mentioned above, we can get more details of the image by analyzing the gradient vector field. Therefore, how to digital the geometric problem becomes the core problem of extracting the skeleton. The topology analysis method will provide us with some new ideas to get more details of the skeleton of the image.

In the topology analysis [8,9], we can obtain the other tangents shapes, even the whole structure of the vector field if we can know in advance some points in vector field and the corresponding tangent. The point with these characteristics are called critical points. Critical point is a collection of points when the vector field is zero, namely critical point exists where all of the component disappear. The tangent nearby indicates that this is a critical point. Therefore, we can further define the skeleton as a collection of critical points in the GVF, namely the skeleton is the set of all the saddle points and nodes in the GVF.

In the vicinity of the critical point (x_0, y_0), the Taylor series expansion of the vector field is [8]:

$$V(x,y) = V(x_0,y_0) + \left(\frac{\partial(u,v)}{\partial(x,y)}\right)_{(x_0,y_0)} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} + O\begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} \tag{3}$$

Here, $O\begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$ represents Taylor series remainder, namely the higher order term; The behavior of GVF is mainly determined by the first order partial derivative of the GVF field near to the critical point in the formula (3). Namely, in formula (4) Jacobian matrix of critical point can be used to characterize the vector field and its nearby vicinity tangent. Meanwhile, the real part of Jacobian matrix eigenvalues's symbols determines the tangent characteristics to attract or repel.

$$\left(\frac{\partial(u,v)}{\partial(x,y)}\right)_{(x_0,y_0)} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}_{(x_0,y_0)} \tag{4}$$

The critical points can be divided into saddle point, node and exclusion point, in which μ and λ are eigenvalues of the Jacobian matrix. In practice, we can get the saddle point and the node's properties by analyzing the characteristics of the critical points, then based on which to look for the saddle point and nodes.In the gradient vector flow field, a new physical quantity flux is defined based on the gradient vector flow field–flux. And flux together with the above topological analysis method act as one

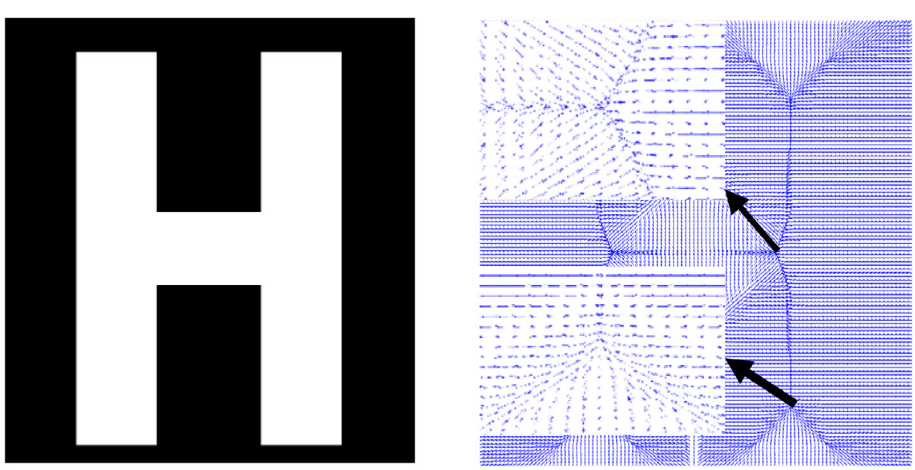

Fig. 1. H model and the GVF (a) H model (b) GVF.