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Slip flow by a variable thickness rotating disk subject to magnetohydrodynamics

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ABSTRACT

Objective of the present study is to determine the characteristics of magnetohydrodynamic flow by a rotating disk having variable thickness. At the fluid–solid interface we consider slip velocity. The governing nonlinear partial differential equations of the problem are converted into a system of nonlinear ordinary differential equations. Obtained series solutions of velocity are convergent. Impact of embedded parameters on fluid flow and skin friction coefficient is graphically presented. It is observed that axial and radial velocities have an opposite impact on the thickness coefficient of disk. Also surface drag force has a direct relationship with Hartman number.

Introduction

Engineering and industrial applications due to rotating surfaces have attracted the attention of scientists and researchers. These applications include air cleaning machines, gas turbines, medical equipment, food processing technology, aerodynamical engineering and in electric-power generating systems. Initial work on rotating disk flow was undertaken by Karman [1]. Ordinary differential equations were obtained from Navier–Stokes equations by using the Von Karman transformation. Subsequently different physical problems were discussed by various researchers. In the internal cooling-air systems of most gas turbines disks rotating at different speeds are found. Heat transfer and flow associated with an air-cooled turbine disk and an adjacent stationary casing were modeled using the rotor–stator system. Bachok et al. [2] examined nanofluid flow due to rotation of a permeable disk. Similarity solution for flow and heat convection from a porous rotating disk was considered by Kendoush [3]. MHD slip flow with variable properties and entropy generation due to rotation of a permeable disk were investigated by Rashidi et al. [4]. Turkyilmazoglu [5] described heat transfer and flow due to rotation of disk with nanoparticles. Sheikholeslami et al. [6] examined nanofluid spraying on an inclined rotating disk for cooling process. Hayat et al. [7] studied partial slip effects in MHD flow due to rotation of a disk with nanoparticles. Mustafa et al. [8] analyzed MHD stagnation point flow of a ferrofluid past a stretchable rotating disk. Xun et al. [9] studied flow and heat transfer of Ostwald-de Waele fluid over a variable thickness rotating disk. Chemical reaction effects in flow of ferrofluid due to a rotating disk were presented by Hayat et al. [10].

There are promising applications in metallurgy, polymer industry, chemistry, engineering and physics due to fluid flow in the presence of a magnetic field. Desired characteristics of the end product are attained in such applications by controlling the rate of heat cooling. The rate of cooling is controlled by magnetic field for an electrically conducting fluid. Physiological fluid applications like blood pump machines and blood plasma are of great importance for MHD flow. Flow configurations under different conditions for MHD flows were considered by numerous researchers. Effects of velocity slip and temperature jump in a porous medium by a shrinking surface with magnetohydrodynamic were studied by Zheng et al. [11]. Analytical and numerical solutions for MHD Falkner-Skan Maxwell fluid flow were presented by Abbasbandy et al. [12]. Turkyilmazoglu [13] examined MHD flow of viscoelastic fluid over a stretching/shrinking surface in three dimensional analysis. Sheikholeslami et al. [14] analyzed radiative flow of nanofluid with magnetohydrodynamics. A numerical study of radiative MHD flow of Al_2O_3–water nanofluid has been studied by Sheikholeslami et al. [15]. Hayat et al. [16] studied Cattaneo–Christov heat flux in

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MHD flow of Oldroyd-B fluid with chemical reaction. Li et al. [17] examined MHD viscoelastic fluid flow and heat transfer by a vertical stretching sheet with Cattaneo–Christov heat flux. Makinde et al. [18] presented MHD Couette-Poiseuille flow of variable viscosity nanofluids in a rotating permeable channel with Hall effects. Numerical study of MHD nanofluid flow and heat transfer past a bidirectional exponentially stretching sheet was considered by Ahmad et al. [19]. Buoyancy effects on the three dimensional MHD stagnation-point flow of a Newtonian fluid were examined by Borrellia et al. [20].

The formation and use of microdevices remain a hotly debated and challenging topic of research by scientists. The small size as well as high efficiency of microdevices-such as microsensors, microvalves and micropumps are some of the advantages of using microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS). Wall slip readily occurs for an array of complex fluids such as emulsions, suspensions, foams and polymer solutions. Also fluids that exhibit boundary slip have important technological applications, such as polishing of artificial heart valves and internal cavities. Many attempts addressing slip flow have been presented to guarantee the performance of such devices. A number of models have been proposed for describing slip that occurs at solid boundaries. A brief description of these models may be found in the work of Rao and Rajagopal [21]. Buscall [22] reported that the importance of studying wall slip has grown substantially. Akbarinia et al. [23] used microchannels to study nanofluids heat transfer enhancement in non-slip and slip flow regimes. Micropolar fluid flow with velocity slip and heat generation (absorption) has been examined by Mahmoud and Waheed [24]. Velocity and thermal slip effects in nanofluid flow have been analyzed by Khan et al. [25]. Mukhopadhyay [26] examined radiative flow over a permeable exponentially stretching sheet with slip effects. Inside a circular microchannel slip flow of alumina/water nanofluid has been considered by Malvandi and Ganji [27]. Nonlinear thermal radiation and slip velocity in MHD three-dimensional nanofluid flow have been studied by Hayat et al. [28]. Slip flow in a microchannel for nanoparticles using lattice Boltzman method has been analyzed by Karimipour et al. [29]. Effect of mass transfer induced velocity slip on heat transfer of viscous gas flows over stretching/shrinking sheet has been presented by Wu [30].

In the past much attention has been given to flow due to a rotating disk with negligible thickness. Our main focus of the present analysis is to study MHD flow of viscous fluid due to a rotating disk with variable thickness. Formulation and analysis is presented when no slip does not remain valid. The technique used for solving the present problem is homotopy analysis method (HAM) [31–38]. Convergent series solutions are obtained. Impacts of pertinent parameters on axial, radial and tangential velocity components and surface drag force are examined.

Formulation

Consider steady, laminar and axisymmetric flow due to a disk rotating with angular velocity Ω about the z-axis. The disk is also stretched with velocity $u_w = ra_1$ where a_1 is the stretching rate constant. We assume that the disk at $z = a\left(\frac{r}{R_0}+1\right)^{-m}$ is not flat where a is the disk thickness coefficient, R_0 is the feature radius and m is the disk thickness index. Slip flow regime is considered for viscous fluid. A magnetic field of strength B_0 is applied in the z-direction. Magnetic Reynolds number is assumed small and thus induced magnetic field is neglected. Electric field is taken absent. The governing equations are as follows:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\nu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2 v}{\rho}, \tag{3}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=\nu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right), \tag{4}$$

with boundary conditions

$$\begin{aligned}&u = ra_1+\lambda_1\tfrac{\partial u}{\partial z},\ v=r\Omega+\lambda_2\tfrac{\partial v}{\partial z},\ w=0 \text{ at } z=a\left(\tfrac{r}{R_0}+1\right)^{-m},\\ &u\to 0,\ v\to 0 \text{ as } z\to\infty,\end{aligned} \tag{5}$$

where u, v and w are velocity components in the direction of r, Θ and z respectively, ν denotes kinematic viscosity, σ the electrical conductivity, ρ the density and λ_1, λ_2 are slip velocity coefficients. Generalized Von Karman transformations are

$$\begin{aligned}&u=r\Omega F(\eta),\ v=r\Omega G(\eta),\ w=R_0\Omega\left(1+\tfrac{r}{R_0}\right)^{-m}\left(\tfrac{\Omega R_0^2\rho}{\mu}\right)^{\frac{-1}{n+1}}H(\eta)\\ &\eta=\tfrac{z}{R_0}\left(1+\tfrac{r}{R_0}\right)^{-m}\left(\tfrac{\Omega R_0^2\rho}{\mu}\right)^{\frac{1}{n+1}}.\end{aligned} \tag{6}$$

Mass conservation law is identically satisfied and Eqs. (2)–(5) become

$$H'+2F+\eta\varepsilon mF'=0, \tag{7}$$

$$\mathrm{Re}^{\frac{1-n}{1+n}}(1+r^*)^{2m}F''-F^2-\eta\varepsilon mFF'+G^2-HF'-MF=0, \tag{8}$$

$$\mathrm{Re}^{\frac{1-n}{1+n}}(1+r^*)^{2m}G''-2FG-\eta\varepsilon mFG'-HG'-MG=0, \tag{9}$$

with boundary conditions

$$\begin{aligned}&H(\alpha)=0,\ F(\alpha)=A+\gamma_1(1+r^*)^mF'(\alpha),\ G(\alpha)\\ &\quad=1+\gamma_2(1+r^*)^mG'(\alpha),\ F(\infty)\to 0,\ G(\infty)\to 0,\end{aligned} \tag{10}$$

where $\varepsilon=\frac{r}{R_0+r}$ is a dimensionless constant, $\mathrm{Re}=\frac{\Omega R_0^2}{\nu}$ is the Reynolds number, $A=\frac{a_1}{\Omega}$ is scaled stretching parameter, $r^*=\frac{r}{R_0}$ is the dimensionless radius, $\alpha=\frac{a}{R_0}\left(\frac{\Omega R_0^2\rho}{\mu}\right)^{\frac{-1}{n+1}}$ is the dimensionless disk thickness coefficient, $\gamma_1=\frac{\lambda_1}{R_0}\left(\frac{\Omega R_0^2\rho}{\mu}\right)^{\frac{1}{n+1}}$ and $\gamma_2=\frac{\lambda_2}{R_0}\left(\frac{\Omega R_0^2\rho}{\mu}\right)^{\frac{1}{n+1}}$ are velocity slip parameters and $M=\frac{\sigma B_0^2}{\rho\Omega}$ is the Hartman number.

We now consider

$$H(\eta-\alpha)=h(\xi),\ F(\eta-\alpha)=f(\xi),\ G(\eta-\alpha)=g(\xi), \tag{11}$$

and thus Eqs. (7)–(10) are reduced to

$$h'+2f+(\xi+\alpha)\varepsilon mf'=0, \tag{12}$$

$$\mathrm{Re}^{\frac{1-n}{1+n}}(1+r^*)^{2m}f''-f^2+g^2-hf'-\varepsilon m(\xi+\alpha)ff'-Mf=0, \tag{13}$$

$$\mathrm{Re}^{\frac{1-n}{1+n}}(1+r^*)^{2m}g''-2fg-hg'-\varepsilon m(\xi+\alpha)fg'\varsigma-Mg=0, \tag{14}$$

$$\begin{aligned}&h(0)=0,\ f(0)=A+\gamma_1(1+r^*)^mf'(0),\ g(0)\\ &\quad=1+\gamma_2(1+r^*)^mg'(0),\ f(\infty)\to 0,\ g(\infty)\to 0.\end{aligned} \tag{15}$$

Here prime denotes the derivative with respect of ξ and h, f and g are axial, radial and tangential velocity profiles respectively.

At the disk the shear stress in radial and tangential directions is τ_{zr} and $\tau_{z\theta}$

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