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Optimal solutions for a bio mathematical model for the evolution of smoking habit

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ABSTRACT

In this study, we apply Variation of Parameter Method (VPM) coupled with an auxiliary parameter to obtain the approximate solutions for the epidemic model for the evolution of smoking habit in a constant population. Convergence of the developed algorithm, namely VPM with an auxiliary parameter is studied. Furthermore, a simple way is considered for obtaining an optimal value of auxiliary parameter via minimizing the total residual error over the domain of problem. Comparison of the obtained results with standard VPM shows that an auxiliary parameter is very feasible and reliable in controlling the convergence of approximate solutions.

Introduction

The study of epidemiological process such as spread of non-fatal diseases in a population has been widely used for the spread of social habits, such as smoking habit [1], obesity epidemics [2]. alcohol consumption [3], or cocaine consumption [4].

In this study, we consider the model for the spread of smoking habit in Spain presented in [1]. In this model, we are using real data for the parameters and for the initial values. We consider a constant population with equal birth and death rates but different from zero, which results in the total number of individuals constant, but those individuals are constantly renewed.

$$
\begin{aligned}
\frac{dx}{dt} &= \mu - (d_0+\mu)x(t) + d_0x^2(t) + (d_f-\beta)x(t)(y(t)+s(t)) + \left(\frac{d_0+d_f}{2}\right)x(t)b(t),\\
\frac{dy}{dt} &= \beta x(t)(y(t)+s(t)) + \rho b(t) + \alpha s(t) - (\gamma+\lambda+\mu+d_f)y(t) + d_0x(t)y(t) + d_fy(t)(y(t)+s(t)) + \left(\frac{d_0+d_f}{2}\right)y(t)b(t),\\
\frac{ds}{dt} &= \gamma y(t) - (\alpha+\delta+\mu+d_f)s(t) + d_0x(t)s(t) + d_fs(t)(y(t)+s(t)) + \left(\frac{d_0+d_f}{2}\right)s(t)b(t),\\
\frac{db}{dt} &= \lambda y(t) + \delta s(t) - \left(\rho+\mu+\frac{d_0+d_f}{2}\right)b(t) + d_0x(t)b(t) + d_fb(t)(y(t)+s(t)) + \left(\frac{d_0+d_f}{2}\right)b^2(t),
\end{aligned}
\tag{1}
$$

The four types of individuals for the population are non-smokers who has never smoked, normal smokers who smoked less than 20 cigarettes per day, excessive smokers who smoked more than 20 cigarettes per day and ex-smokers who had smoked in the past are denoted respectively by X, Y, S and B. The graphical representation of the proposed model is shown in Fig. 1.

We have the following system of differential equations (see [1] for detail)

with the initial conditions

$$x(0)=0.5045,\; y(0)=0.2059,\; s(0)=0.1559,\; b(0)=0.1337. \tag{2}$$

where the new scaled variables are:

$$x=\frac{X}{P},\; y=\frac{Y}{P},\; s=\frac{S}{P},\; b=\frac{B}{P}$$

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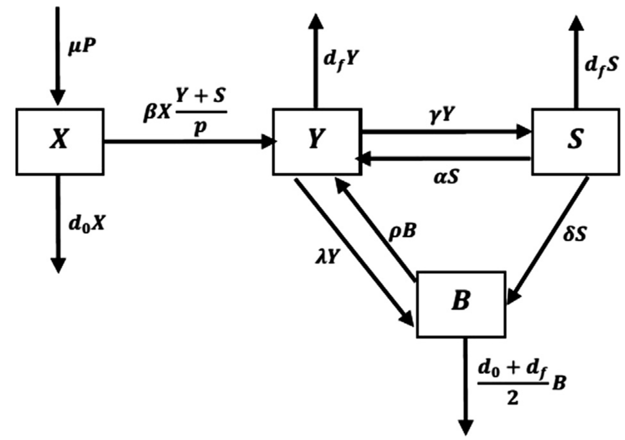

Fig. 1. Flow diagram of smoking model.

Table 1
Values of parameters with their descriptions.

Notation	Value	Description of parameter
μ	0.01	Natural birth rate
d_0	0.0087	Natural death rate
d_f	0.0132	Increased death rate due to smoking disease
β	0.0381	Transmission rate to adopt smoking habit
ρ	0.0425	Rate at which an ex-smoker returns to smoking
α	0.1244	Rate at which an excessive smoker becomes a normal smoker
γ	0.1175	Rate at which a normal smoker becomes an excessive smoker
λ	0.0498	Rate of stopping smoking for normal smokers
δ	0.0498	Rate of stopping smoking for excessive smokers

while P denotes the total population.

The system (1) contains several parameters whose numerical values were obtained from [1]. The numerical values and descriptions of the parameters are summarized in Table 1.

Since the total number of individuals has been normalized to unity, we have the following equation

$$x(t) + y(t) + s(t) + b(t) = 1. \tag{3}$$

Several analytical algorithms like Variation of Parameter Method (VPM) [5–12], Differential Transform Method (DTM) [13–15], Homotopy Perturbation Method (HPM) [16,17], Homotopy Analysis Method (HAM) [18], Adomian Decomposition Method (ADM) [19,20] and Optimal Homotopy Asymptotic Method (OHAM) [21,22] are powerful tool to obtain approximate solutions of differential equations. Among the above mentioned algorithms Ma's VPM is highlighted due to its versatility and simplicity to solve a wide class of nonlinear problems [8–11,23,24]. The main advantage of method is that, it does not rely on linearization, discretization, perturbation, restrictive assumptions and is free from calculation of Adomian's polynomials. The results obtained are completely reliable with the proposed algorithm and are very encouraging

The aim of present study is to concentrate on the application of Variation of Parameters Method (VPM) [5–12] coupled with an auxiliary parameter to a model for the evolution of smoking habit given in (1). The main advantage of developed algorithm is its ability in providing the better information of continuous approximate solution over the given time interval. In the modified algorithm the total residual error is defined to choose an optimal value of auxiliary parameter. Convergence of modified scheme is also shown and discussed in detail. Numerical results obtained by the proposed algorithm are very effective, reliable and encouraging as compared with the standard VPM.

Variation of Parameters Method (VPM)

To convey the basic step of VPM for differential equations, we consider a general nonlinear ordinary differential equation in operator form as follows:

$$Hf(\eta) = Lf(\eta) + Rf(\eta) + Nf(\eta) + g(\eta) = 0 \tag{4}$$

where L represents the higher order linear operator, R shows a linear operator of order less then L, N is a nonlinear operator, and g is an inhomogeneous term. The variation of parameters method [5–12] provides the general iterative scheme for Eq. (4) as:

$$f_0(\eta) = \sum_{i=0}^{m} \frac{\eta^i f^i(0)}{i!} \quad \text{is an initial approximation}$$
$$f_{n+1}(\eta) = f_0(\eta) + \int_0^{\eta} \lambda(\eta,\zeta)(-Rf_n(\zeta) - Nf_n(\zeta) - g(\zeta))d\zeta, \quad n \geqslant 0 \tag{5}$$

where m is the order of given differential equation and $\lambda(x,\zeta)$ is multiplier which can be determined with the help of Wronskian technique

$$\lambda(\eta,\zeta) = \sum_{i=1}^{m} \frac{(-1)^{i-1}\zeta^{i-1}\eta^{m-i}}{(i-1)!(m-i)!} = \frac{(\eta-\zeta)^{m-1}}{(m-1)!}, \tag{6}$$

Consequently, an exact solution can be obtained when n approaches to infinity:

$$f(\eta) = \lim_{n\to\infty} f_n(\eta). \tag{7}$$

Variation of parameters method with an auxiliary parameter (OVPM)

An unknown auxiliary parameter κ can be inserted into the variation of parameter method. Eq. (4) can be easily written in the following form:

$$Lf(\eta) = Lf(\eta) + \kappa Hf(\eta). \tag{8}$$

According to variation of parameters method, we can construct the following iterative scheme for Eq. (8):

$$f_0(\eta) = \sum_{i=0}^{m} \frac{\eta^i f^i(0)}{i!} \quad \text{is an initial approximation}$$
$$f_1(\eta,\kappa) = f_0(\eta) + \int_0^{\eta} \lambda(\eta,\zeta)(Lf_0(\zeta) + \kappa Hf_0(\zeta))d\zeta,$$
$$f_{n+1}(\eta,\kappa) = f_0(\eta) + \int_0^{\eta} \lambda(\eta,\zeta)(Lf_n(\zeta,\kappa) + \kappa Hf_n(\zeta,\kappa))d\zeta, \quad n \geqslant 1. \tag{9}$$

The successive approximation $f_n(\eta,\kappa), n \geqslant 1$ contain an auxiliary parameter κ, which is used to control and adjust the convergence of approximate solution that can be determined optimally by minimizing the norm 2 of the residual error [25–27]. In fact, the suggested algorithm is simple, reliable, and effective and is accurately approximate the solution in a bigger domain.

Convergence analysis

In this section, convergence of modified scheme is presented according to alternative approach of this scheme.

Lemma 4.1. Let L, defined in (4) be as, $L = \frac{d^m}{d\eta^m}$,where m is the order of differential equation and $\lambda(\eta,\zeta) = \frac{(\eta-\zeta)^{m-1}}{(m-1)!}$, is a multiplier. If $G(\eta)$ be a function from Hilbert space H to H, then

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