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Hall and Joule heating effects on peristaltic flow of Powell–Eyring liquid in an inclined symmetric channel

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ABSTRACT

This article is intended to investigate the influence of Hall current on peristaltic transport of conducting Eyring–Powell fluid in an inclined symmetric channel. Energy equation is modeled by taking Joule heating effect into consideration. Velocity and thermal slip conditions are imposed. Lubrication approximation is considered for the analysis. Fundamental equations are non-linear due to fluid parameter *A*. Regular perturbation technique is employed to find the solution of systems of equations. The key roles of different embedded parameters on velocity, temperature and heat transfer coefficient in the problem are discussed graphically. Trapping phenomenon is analyzed carefully.

Introduction

Investigation regarding the flow of non-Newtonian fluid cannot be overlooked due to its extensive applications in variety of field like physiology, engineering and industry. No doubt various constitutive relations are suggested for the flow description of such fluids diverse characteristics. Some recent researchers are even now engaged for the flow analysis of such fluids. In connection with peristalsis, the non-Newtonian fluids gained much attention due to their various applications in physiological and industrial processes. Spontaneous compressing and relaxing movement along the walls of tabular structures is termed as peristalsis. Digestive tract, blood flow in lymphatic transport are few examples that can be observed within human body. The phenomenon is also involved in designing many devices like dialysis machine, heart lung machine and blood pump machine to blood pump during surgical processes. Some worms also use this phenomenon for their locomotion. Pioneering studies in this direction were done by Latham [1], Shapiro et at. [2] and Lew et al. [3]. After these attempts the investigators analyzed the peristaltic flow of Newtonian and non-Newtonian fluids under different flow situations [4–10]. Heat transfer also has a vital role in peristaltic flows especially blood flows. Heat conduction in tissues, convective heat transfer during blood flow from pores of tissue, radiative heat transfer between environment and surface, food processing and vasodilation are some main applications of heat transfer. Oxygenation and hemodialysis are the processes involving heat transfer in connection with peristalsis. Recent attempts on peristaltic flow with heat transfer effects can be visualized by Refs. [11–20].

Magnetic field has gained significance due to its variety of applications in biomedical engineering and industry. Power generators, electrostatic precipitation, purification of molten metal from nonmetallic inclusions etc. are some processes that deals with magnetic field. The shear rate of less than $100\,s^{-1}$ for blood flow shows the model for MHD peristaltic flows in coronary arteries [21]. MHD may also be used to control the blood flow during cardiac surgeries from stenosed arteries. Hall effects cannot be ignored when strong magnetic field is considered. Representative studies in this direction can be consulted by the Refs.[22–31].

The problems studying thin films, rarefied fluid, fluid motion inside human body and polishing of artificial heart values etc. do not follow no-slip boundary condition. Experimental investigations show that slippage can occur in non-Newtonian fluids. Moreover, many physiological systems are neither horizontal nor vertical but show inclination with axis (see Refs. [32–36]). Therefore, aim of the present study is to investigate the peristaltic flow of Powell–Eyring liquid in an inclined symmetric channel. Heat transfer is studied in the presence of Joule heating. Problem is formulated by taking partial slip effects into account. Nonlinear equations are simplified by adopting lubrication approach. Perturbation is

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employed to find the solution of stream function, velocity, temperature and heat transfer coefficient. Results are analyzed via graphs.

Formulation

Here we assume the two-dimensional electrically conducting non-Newtonian incompressible fluid in an inclined symmetric channel having width $2d$ (see Fig. 1). We consider Cartesian coordinates (x, y) in such a way that wave propagates in x-direction and y-axis is taken transverse to it. The walls of channel are assumed compliant. Also the channel is inclined at angle α. Strong magnetic field $(0, 0, B_0)$ is applied. Hall and Joule heating contributions are retained. Peristaltic waves propagate with constant speed c and wavelength λ along channel walls. The structure of wall geometry is described as:

$$y = \pm\eta(x,t) = \pm\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right]. \tag{1}$$

Here t is the time, a the wave amplitude and $\pm\eta$ the displacements of upper and lower walls respectively.

The Cauchy stress tensor (τ) for Eyring–Powell fluid is Refs. [19,23]

$$\tau = -p\mathbf{I} + \mathbf{S}, \tag{2}$$

$$\mathbf{S} = \left[\mu + \frac{1}{\beta\dot{\gamma}}\sinh^{-1}\left(\frac{\dot{\gamma}}{c_1}\right)\right]\mathbf{A}_1, \tag{3}$$

$$\dot{\gamma} = \sqrt{\frac{1}{2}\Pi}, \tag{4}$$

$$\Pi = tr(\mathbf{A}_1^2), \mathbf{A}_1 = \text{grad}\,\mathbf{V} + (\text{grad}\,\mathbf{V})^*. \tag{5}$$

Here $\mathbf{S}$ designates the extra stress tensor, $\mathbf{I}$ the identity tensor, β and c_1 the material parameters of Powell–Eyring fluid and μ the dynamic viscosity. The term $\sinh^{-1}$ is

$$\sinh^{-1}\left(\frac{\dot{\gamma}}{c_1}\right) = \frac{\dot{\gamma}}{c_1} - \frac{\dot{\gamma}^3}{6c_1^3}, \frac{\dot{\gamma}^5}{c_1^5} \ll 1. \tag{6}$$

The generalized Ohms law with Hall effects is written as:

$$\mathbf{J} = \sigma\left[\mathbf{V} \times \mathbf{B} - \frac{1}{en}(\mathbf{J} \times \mathbf{B})\right], \tag{7}$$

$$\mathbf{J} \times \mathbf{B} = \frac{-\sigma B_0^2}{1 + m^2}[(u - mv), (v + mu), 0], \tag{8}$$

in which $\mathbf{J}$ characterizes the current density, $\mathbf{V}$ the velocity field, $\mathbf{B}$ the applied magnetic field, σ the electrical conductivity, n the number density of electron, e the electric charge, u and v the velocity components in x and y directions respectively, B_0 the magnetic field strength and $m\left(= \frac{\sigma B_0}{en}\right)$ the Hall parameter. The fundamental flow equations are

$$div\,\mathbf{V} = 0, \tag{9}$$

$$\rho\frac{d\mathbf{V}}{dt} = div\,\mathbf{T} + \mathbf{J} \times \mathbf{B} + \rho\mathbf{g}, \tag{10}$$

$$\rho C_P\frac{d\mathbf{T}}{dt} = \mathbf{T}.\mathbf{L} + \kappa\nabla^2\mathbf{T} + \frac{\mathbf{J}.\mathbf{J}}{\sigma}, \tag{11}$$

in which ρ is the fluid density, κ the thermal conductivity and C_p the specific heat.

The two dimensional fundamental flow equations after using Eqs. (2)–(8) in Eqs. (9)–(11) can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{12}$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} + u\frac{\partial v}{\partial x}\right) = -\frac{\partial p}{\partial y} + \frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x} + \frac{\sigma B_0^2}{1 + m^2}(v + mu) + \rho g\cos\alpha, \tag{13}$$

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} + \frac{\sigma B_0^2}{1 + m^2}(u - mv) + \rho g\sin\alpha, \tag{14}$$

$$\rho C_p\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial y} + u\frac{\partial}{\partial x}\right)T = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + (S_{yy} - S_{xx})\frac{\partial v}{\partial y} + S_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\sigma B_0^2}{1 + m^2}(u^2 + v^2), \tag{15}$$

where $p, S_{ij}\ (i,j = x,y), g$ and T signify the pressure, the components of extra stress tensor, the gravity and temperature respectively.

The slip conditions for velocity and temperature at the walls are:

$$u \pm \gamma S_{xy} = 0 \quad \text{at} \quad y = \pm\eta, \tag{16}$$

$$T \pm \beta_1\frac{\partial T}{\partial y} = T_0 \text{ at } y = \pm\eta. \tag{17}$$

Flexible walls can be characterized by

$$\left[-\tau\frac{\partial^3}{\partial x^3} + m_1\frac{\partial^3}{\partial x\partial t^2} + d\frac{\partial^2}{\partial t\partial x}\right]\eta = \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x}\right) - \frac{\sigma B_0^2}{1 + m^2}(u - mv) + \rho g\sin\alpha \text{ at } r = \pm\eta. \tag{18}$$

In the above expressions T_0 is the temperature at the upper and lower walls, τ is the elastic tension, m_1 the mass per unit area and d the coefficient of viscous damping.

Dimensionless parameters are:

$$x^* = \frac{x}{\lambda},\quad y^* = \frac{y}{d},\quad u^* = \frac{u}{c},\quad v^* = \frac{v}{c},\quad t^* = \frac{ct}{\lambda},$$
$$\eta^* = \frac{\eta}{d},\quad p^* = \frac{d^2 p}{c\lambda\mu},\quad \gamma^* = \frac{\gamma}{d},\quad \beta_1^* = \frac{\beta_1}{d}, \tag{19}$$
$$\theta = \frac{T - T_0}{T_0},\quad S_{ij}^* = \frac{dS_{ij}}{c\mu},\quad \psi^* = \frac{\psi}{cd}.$$

By using dimensionless variables, Eqs. (12)–(15) become:

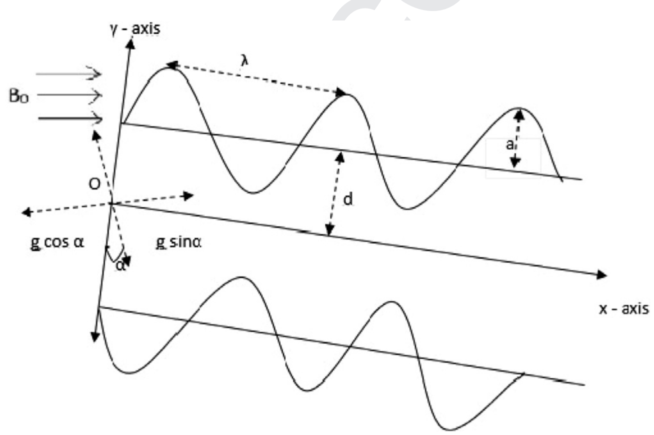

Fig. 1. Geometry of the problem.