



Review

Accurate characterization of weak neutron fields by using a Bayesian approach



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ABSTRACT

A Bayesian analysis of data derived from neutron spectrometric measurements provides the advantage of determining rigorously integral physical quantities characterizing the neutron field and their respective related uncertainties.

The first and essential step in a Bayesian approach is the parameterization of the investigated neutron spectrum.

The aim of this paper is to investigate the sensitivity of the Bayesian results, mainly the neutron dose $H^*(10)$ required for radiation protection purposes and its correlated uncertainty, to the selected neutron spectrum parameterization.

1. Introduction

Neutron environmental measurements are often carried out with passive detectors such as Bonner Spheres Spectrometer or threshold detectors. These types of detectors do not provide a direct measurement of the neutron spectrum. Different mathematical methods, including the Bayesian inference, should be used to deduce the investigated spectrum by unfolding the data derived from the used spectrometers.

Some works have suggested that a Bayesian analysis of neutron spectrometric measurements (Reginatto, 2006) is better suited for quantifying the uncertainties of the derived integral quantities as neutron doses.

In neutron spectrometry, a Bayesian analysis begins with the parameterization of the investigated spectrum. This operation is often difficult to carry out, particularly when the shape of the measured spectrum is nearly known or in the presence of a complex spectrum (the presence of many random structures makes the choice of the spectrum parameterization difficult).

The main objective of this paper is to study the sensitivity of the Bayesian results, especially quantities required in radiation protection applications such as the neutron equivalent ambient dose $H^*(10)$. Some selection arguments for the choice of the most appropriate neutron spectrum model are proposed.

Furthermore, the sensitivity of the obtained Bayesian results to the assumptions attached to the prior probability distributions, describing the investigated spectrum parameters, is studied.

The proposed analysis method is suited for the characterization and

for radiation protection applications of environmental neutron field in the case where the neutron spectrum shape is known and many parameterizations can be proposed that makes the choice difficult. This situation is often encountered in the characterization of parasite neutrons fields, generated by high-energy gamma rays, around nuclear installations such as linear medical accelerators.

As an example, we consider here, the analysis of neutron measurements derived from a set of thick threshold detectors by the Bayesian approach. This low-volume spectrometer, known for its robustness and cheapness, is quite suited for environmental neutron measurements, particularly in hazardous environments and in areas of difficult access, such as at the head of a linear medical accelerator.

In a previous study (Medkour et al., 2012), a thick threshold spectrometer was successfully tested to characterize the neutron field near an Am-Be source of low activity (~ 37 Gbq) at the Nuclear Research Center of Algiers (CRNA).

In this work, the studied Am-Be spectrum is first modeled by one Gaussian, then by two Gaussians, and finally by a second-degree polynomial. In each case, the model parameters are determined by a Bayesian inference, and the ambient dose value is derived. The selection of the most reliable model to describe the source spectrum is discussed.

2. Bayesian analysis of thick threshold detectors measurements

The measured reaction rate N_k (in Bq per atom) of each reaction induced in a threshold detector k ($k=1, \dots, n$ where n is the number of

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activated foils) can be approximated by its predicted value as reported in Medkour et al. (2012):

$$N_k \approx \int_0^\infty R_k(E)\phi(E) dE \tag{1}$$

where $\phi(E)$ is the differential neutron energy spectrum to which the detectors are exposed.

$R_k(E)$ is the response function per unit of neutron fluence of the k_{th} detector to neutrons of energy E , evaluated as the product of the cross section of the reaction induced in the detector by a geometric factor (Medkour et al., 2012).

The neutron spectrum can then be determined by unfolding of Eq. (1). This equation is an integral equation of a Fredholm type of the first kind and can be expressed for the computational purposes in the discrete form (Matzke, 2003):

$$N_k \approx \sum_{l=1}^m R_{kl}\phi_l \tag{2}$$

where $l=1, \dots, m$ (m is the energy groups number),

$\phi_l = \int_{E_l}^{E_{l+1}} \phi(E)dE$ is the neutron fluence in the interval E_l, E_{l+1} , and.

R_{kl} is the response function of the k_{th} detector in the interval E_l, E_{l+1} .

Unfolding algorithms based on different standard methods (Matzke, 2003) such as least square, iterative, and maximum entropy, reported in detail in Medkour et al., 2012, have been developed to resolve Eq. (2). As the number of detectors n is usually less than the number of unknown energy bins m , some difficulties have been encountered in using these methods. A Bayesian procedure was then applied to determine the Am-Be neutron spectrum and its correlated dose and dose-uncertainty.

The Bayesian approach used in this study starts by modeling the studied neutron spectrum. The discretized neutron spectrum ϕ_l , given in Eq. (2), is modeled in terms of a set of physical parameters α_i defining a vectorial parameter $\vec{\alpha}$ attached to ϕ_l , such as $\phi_l = \phi_l(E_i, \vec{\alpha})$.

We define $P(N_k|\vec{\alpha}, I)$ as the probability that the data N_k would have measured given the vectorial parameter $\vec{\alpha}$ and the available information I about the studied spectrum before the experiment. This probability can be approached by a normal distribution with mean value $\sum_l R_{kl}\phi_l$ (Eq. (2)) and variance $\sigma_{N_k}^2$ (σ_{N_k} is the N_k estimated uncertainty):

$$P(N_k|\vec{\alpha}, I) = \exp\left[-\frac{1}{2}\left[\frac{N_k - \sum_l R_{kl}\phi_l}{\sigma_{N_k}}\right]^2\right] \tag{3}$$

If we note \vec{N} the vector of the measured data N_k , the probability $P(\vec{N}|\vec{\alpha}, I)$ for measuring \vec{N} , given the α_i parameters and the information I , is simply the product of the $P(N_k|\vec{\alpha}, I)$ (Reginatto, 2006):

$$P(\vec{N}|\vec{\alpha}, I) = \prod_k P(N_k|\vec{\alpha}, I) = \exp\left[-\frac{1}{2}\chi^2\right] \tag{4}$$

where χ^2 , the chi squared statistic, is given, using Eq. (3), as follows:

$$\chi^2 = \sum_k \left[\frac{N_k - \sum_l R_{kl}\phi_l}{\sigma_{N_k}}\right]^2 \tag{5}$$

The posterior probability $P(\vec{\alpha}|\vec{N}, I)$ of the α_i parameter is evaluated by applying the Bayes' theorem (Gregory, 2004; Sivia et al., 2006) as the product of the prior probability $P(\vec{\alpha}, I)$ assigned to the α_i parameters and the probability $P(\vec{N}|\vec{\alpha}, I)$:

$$P(\vec{\alpha}|\vec{N}, I) \propto P(\vec{N}|\vec{\alpha}, I)P(\vec{\alpha}, I) \tag{6}$$

Uniform prior probability is often assigned to each α_i in its definition interval.

The α_i posterior probability distribution is calculated using the marginal equation as described in Reginatto (2006) and Reginatto and Zimbal (2008):

$$P(\alpha_i|\vec{N}, I) \propto \int \exp\left[-\frac{1}{2}\chi^2\right] \prod_{j \neq i} (P(\alpha_j)d\alpha_j) \tag{7}$$

The Bayesian analysis leads to a probability distribution for each α_i used in the neutron spectrum modeling. A good estimate of each α_i (optimal value) is given by the point where its probability distribution peaks. An estimate of the spectrum $\phi_l(E_i, \vec{\alpha})$ is calculated using the optimal α_i parameters values.

The ambient equivalent dose $H^*(10)$, an important physical quantity in radiation protection dosimetry, will be derived from this analysis. Its posterior probability can be calculated as follows (Reginatto, 2006; Reginatto and Zimbal, 2008):

$$P(H^*(10)|\vec{N}, I) \propto \int \delta(H^*(10) - \sum_l h_l \phi_l(E_l, \vec{\alpha})) \exp\left[-\frac{1}{2}\chi^2\right] P(\vec{\alpha}, I) d\vec{\alpha} \tag{8}$$

where δ represents the delta function and the dose conversion coefficients values h_l are given by the International Commission on Radiological Protection 74(ICRP 74, 1997).

The best estimate value of $H^*(10)$ is given by the value where its probability distribution peaks and the ambient dose probability distribution width (full width at half maximum (FWHM)) furnishes a good assessment of the neutron ambient dose uncertainty.

The Bayesian approach described above depends, among others, on the selected neutron spectrum parameterization and on the prior information. A study of the sensitivity of the deduced results to these parameters has been performed, particularly by analyzing the value of the neutron equivalent ambient dose $H^*(10)$.

3. Sensitivity analysis for the neutron spectrum parameterization

In the present study, the scarcely available experimental data (only six measured detector responses) limit the choice of the model spectrum and the number of its free parameters.

To check the sensitivity of the $H^*(10)$ Bayesian results to the selected neutron spectrum parameterization, the studied spectrum was approached first by one Gaussian, then by a sum of two Gaussians, and finally by a second-degree polynomial, knowing that each mathematical curve can be approached by a sum of Gaussians or by a serial polynomial or by a combination of both functions.

3.1. Neutron spectrum modeled by one Gaussian

The investigated spectrum was approached by a Gaussian characterized by magnitude A_0 , width σ_0 , and mean energy E_0 :

$$\phi_l = \phi_l(E_i, \vec{\alpha}) = A_0 \exp\left(-\frac{(E_i - E_0)^2}{2\sigma_0^2}\right) \tag{9}$$

where $\vec{\alpha} \equiv A_0, E_0$, and σ_0 represent the model parameters to be determined.

The probability densities of the α_i parameters are calculated using the marginal equations given, according to Eq. (7), by:

$$\begin{aligned} P(A_0|\vec{N}, I) &\propto \int P(A_0, E_0, \sigma_0|I)P(\vec{N}|A_0, E_0, \sigma_0, I)dE_0d\sigma_0 \\ P(E_0|\vec{N}, I) &\propto \int P(A_0, E_0, \sigma_0|I)P(\vec{N}|A_0, E_0, \sigma_0, I)dA_0d\sigma_0 \\ P(\sigma_0|\vec{N}, I) &\propto \int P(A_0, E_0, \sigma_0|I)P(\vec{N}|A_0, E_0, \sigma_0, I)dA_0dE_0 \end{aligned} \tag{10}$$

The integration is made by assuming a lower bound α_{imin} and an upper bound α_{imax} for each α_i value. These bounds are selected so that they would not induce a cutoff in the posterior probability density. This is checked by inspecting the shape obtained for each parameter posterior probability. These integrals were first calculated by assuming that the prior probability $P(A_0, E_0, \sigma_0|I)$ is uniform with respect to A_0, E_0 , and $ln\sigma_0$.

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