Physica Medica 42 (2017) 162–173

Contents lists available at ScienceDirect

Physica Medica

journal homepage: www.elsevier.com/locate/ejmp

Original paper

Medical image segmentation based on level set and isoperimetric constraint

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ARTICLE INFO

Keywords:
Lesion segmentation
Level set
Isoperimetric constraint
Compact shapes
Blurred or/and partially missing boundaries

ABSTRACT

Level set based methods are being increasingly used in image segmentation. In these methods, various shape constraints can be incorporated into the energy functionals to obtain the desired shapes of the contours represented by their zero level sets of functions. Motivated by the isoperimetric inequality in differential geometry, we propose a segmentation method in which the isoperimetric constrain is integrated into a level set framework to penalize the ratio of its squared perimeter to its enclosed area of an active contour. The new model can ensure the compactness of segmenting objects and complete missing or/and blurred parts of their boundaries simultaneously. The isoperimetric shape constraint is free of explicit expressions of shapes and scale-invariant. As a result, the proposed method can handle various objects with different scales and does not need to estimate parameters of shapes. Our method can segment lesions with blurred or/and partially missing boundaries in ultrasound, Computed Tomography (CT) and Magnetic Resonance (MR) images efficiently. Quantitative evaluation also confirms that the proposed method can provide more accurate segmentation than two well-known level set methods. Therefore, our proposed method shows potential of accurate segmentation of lesions for applying in diagnoses and surgical planning.

1. Introduction

Images segmentation performs an important function in medical image processing and analysis [1–4]. However, there are challenges in segmentation due to the presence of noise, low contrast, inhomogeneity, blurred or/and partially missing boundaries and image artifacts in medical images [5–7]. To overcome these difficulties, prior knowledge, such as shape priors [8–13], texture priors [8], appearances priors [14], and statistic priors [15–18], has been used to segment lesions and organs in medical image segmentation.

Level set based methods are powerful and efficient methodologies for image segmentation [19,20]. In particular, they have been successfully applied in the segmentation of medical images [16,18,21–23]. In these methods, a contour is represented as the zero level set of a function. The desired shape of the contour can be controlled by a regularization term (e.g. a shape prior constraint) in the energy functional [24].

The shape prior is a kind of commonly used prior knowledge for segmentation. There are two main approaches to obtain various kinds of shape priors, i.e. training and analytical expressions of shapes. Chen et al. [25] proposed a method to implement in cardiac ultrasound images and functional MR images for corpus callosum segmentation by incorporating a shape prior into a geometric active contour model. In their method, a large number of samples were used in training to obtain a proper shape prior. Leventon et al. [26] employed a trained curvature as prior information for segmenting joint and corpus callosum in MR images. Although the trained shape priors significantly facilitate segmentation, the procedure of training is time consuming. In addition, the final segmentation results are affected by the number and selection of trained samples.

The analytic expression of shapes, such as the equations of circles and ellipses, can be implemented as shape constrains. Because the variations for shapes of objects should be considered in segmentation, shape prior expressions are usually defined in terms of transformations from given shape templates [25]. In this case, the boundaries of objects are obtained by optimizing the parameters of shape transformations including scales, rotations and shifts, so that transformed templates can better fit the real boundaries of objects. Acton et al. [24] proposed a novel method to track the movement of leukocytes and to locate the target cells by using the constraints of shape and size. They used ellipses as shape priors, and five parameters were used to control the shape energy term.

In this paper, a novel shape constraint is proposed for handling a type of shapes, which are called compact shapes. The new shape

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http://dx.doi.org/10.1016/j.ejmp.2017.09.123
Received 2 March 2017; Received in revised form 16 August 2017; Accepted 13 September 2017
1120-1797/

constraint does not rely on any trained prior or parameter selection and estimation, and therefore has good usability in applications. Many lesions and organs (e.g. most renal carcinomas, cysts, breast cancers, nodules as well as cells, kidneys and prostates) in medical images exhibit compact shapes that are quasi-circles, quasi-ellipses and their variants. For instance, the nodular type of hepatic cellular cancer (HCC) regions that are divided into three categories according to their morphological appearances [27] can be regarded as compact shapes. The proposed segmentation method based on our novel shape constraint can therefore be widely used in clinical applications. Accurate segmentation of the organs and lesions in medical images is of great significance. The segmentation results provide precise measurements of these regions, which can be used in the subsequent processes including diagnoses, analysis, and therapy planning. Traditional segmentation is achieved manually. However, this labor-intensive work consumes a lot of time. Moreover, the accuracies of the results are largely depend on the experiences of the operators. Additionally, the nonrepeatability of the results brings inconveniences for the subsequent procedures. In order to reduce the workload and strengthen clinical applications, accurate and robust segmentation methods are necessary in medical image processing. For example, an HCC with a diameter less than 3 cm that is considered small [28], can be effectively cured by percutaneous ethanol injection instead of surgical resection that would be necessary for bigger HCCs [29]. Moreover, precise segmentation results can delineate the boundaries of lesions, which provide the accurate target regions for radiofrequency ablation and radiation therapy. The accurate segmentation results can also be used in comparison of the preoperative and postoperative regions, and the therapeutic evaluation can be acquired precisely.

The proposed shape constraint, called the isoperimetric constraint, is motivated by the well-known isoperimetric inequality in differential geometry. This constraint is based on the ratio of its squared perimeter to the enclosed area of an active contour, and acts as an efficient regularization term. The isoperimetric constraint in the proposed model is integrated into a level set based segmentation scheme. The new model has advantages in three aspects. Firstly, under the isoperimetric constraint, the active contour tends to move to the boundary of a compact shape, which is beneficial for segmenting some lesions and organs in medical images. Moreover, this function is established by the by controlling the ratio of square length to the area, under this circumstance, the smoothness of the contour are ensured. Secondly, the proposed model has an inherent mechanism of completing the missing or/and blurred parts of the boundaries, because the isoperimetric constraint effectively controls the perimeter and the area of a shape at the same time. In other words, the constraint forces the active contour to enclose a compact region and therefore to draw up the objects that even have blurred or/and partially missing boundaries. Thirdly, the isoperimetric constraint is scale invariant. This fact means that the proposed model can handle objects with different sizes and is free of estimating the parameters of shapes.

The proposed model is validated by experiments of segmenting various objects including lesions, organs and structures in different medical images that suffer from noises, inhomogeneities, and blurred or/and partially missing boundaries. In addition, the proposed method is compared with other two well-known level set methods. The average precision and the average Dice coefficient of the proposed method in both ultrasound images and CT images are higher than other two efficient segmentation methods. Experimental results and quantitative analyses indicate that the proposed method can provide the best segmentation results among all the tested methods.

2. Problem formulation

In image segmentation, the length of a contour is one of the most commonly used regularization terms for level set methods. Among all of these methods, the classical Chan-Vese (CV) model [30] is a typical example, in which the arc length of the active contour is adopted as a regularization term. The general form of the CV model is represented as follows:

$$F(C) = E_{data} + Length(C), \tag{1}$$

where C is the active contour; E_{data} is the data term (the fitting term) that forces the contour to close the real boundaries of objects; and the length term (the regularization term) $Length(C)$ controls the smoothness of the active contour. However, this length term may be unable to maintain enough properties for segmenting objects in some complex situations. For instance, in a low-contrast image that is also contaminated by heavy noises, an object is always confused by the background nearby. To tackle such an image, a regularization term with more prior information is needed.

In recent years, various shape constraints are employed as effective regularization terms. Motivated by the isoperimetric inequality in differential geometry, we propose a novel shape constraint and integrate it into a level set framework to efficiently segment compact regions with blurred or/and partially missing boundaries in medical images. The isoperimetric inequality for any bounded Lipschitz domain $\Omega \in R^n, n \geqslant 2$, can be represented as follows [31]:

$$\frac{|\partial\Omega|}{|\Omega|^{\frac{n-1}{n}}} \geqslant n^{\frac{n-1}{n}} C_{n-1}^{\frac{1}{n}}, \tag{2}$$

where $C_{n-1} = \frac{2\pi^{n/2}}{\Gamma(n/2)}$ is called the isoperimetric constant; $\partial\Omega$ is the boundary of the domain Ω and of Lipschitz; and $|\partial\Omega|$ and $|\Omega|$ are the surface measure and volume measure of $\partial\Omega$ and Ω, respectively.

As is well known, in two-dimensional (2D) cases, the shape of Ω is close to a round when the ratio $\frac{|\partial\Omega|}{|\Omega|^{1/2}}$ is close to the isoperimetric constant $2C^{1/2}$. For two regions Ω_1 and Ω_2, if $\frac{|\partial\Omega_1|}{|\Omega_1|^{1/2}} < \frac{|\partial\Omega_2|}{|\Omega_2|^{1/2}}, \frac{|\partial\Omega_1|}{|\Omega_1|^{1/2}}$ is closer to the isoperimetric constant, which means that Ω_1 can be considered to be more compact than Ω_2. In other words, small value of the ratio of a shape indicates that it appears relatively compact. Based on this observation, we calculate the value of $(|\partial\Omega|^2/4\pi|\Omega|)^p$ (also written as $(L^2/4\pi A)^p$) called the $AP-ratio$ to quantitatively characterize compactness. In order to better illustrate the compactness characterization, we calculate the $AP-ratio$ with different values of the parameter p for a series of ellipses with gradual changes of long axis/short axis ratios. As seen in Fig. 1, the $AP-ratio$ rises as the 'long axis/short axis' ratio increases. In addition, the comparison between Fig. 1(a) and (b) indicates that a larger value of the parameter p leads to a faster increase of the $AP-ratio$.

The concept of compactness has been proposed as the prior knowledge in some previous segmentation models. Veksler et al. [32] adopted the ratio of the perimeter to the area ($\frac{L}{A}$) of an object to define its compactness in their segmentation model. Grady et al. [33] employed this compactness concept to describe the shape priors and segmented images by minimizing the ratio $h = \frac{L}{A}$ (in two-dimensional space) under a graph partitioning framework. This graph based method has been used to segment heart chambers in cardiovascular imaging [34], aortic cross-section [35], and unknown objects for baggage security [36]. Koepfler et al. [37] used the similar description of compactness for segmentation. In order to realize the segmentation by region merging, they proposed a merging criterion based on the isoperimetric inequality to eliminate small regions and thin regions.

In this paper, we use the $AP-ratio$: $(L^2/4\pi A)^p$ to describe the compactness of a shape. The $AP-ratio$ is different from the ratio of L/A. The $AP-ratio$ is scale-invariance, for example, different sizes of circles have the same value of the $AP-ratios$, this property is reasonable and important because of the fact that even the same lesions and organs may present different sizes in different times. On the contrary, the value of the ratio L/A decreases as the radii of a circle increases. As a shape constraint, this $AP-ratio$ is embedded into a level set framework. The smoothness and the compactness of the segmented regions are guaranteed by the proposed constraint.