ARTICLE IN PRESS

Radiation Physics and Chemistry ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Radiation Physics and Chemistry

journal homepage: www.elsevier.com/locate/radphyschem

First principles pulse pile-up balance equation and fast deterministic solution

Lorenzo Sabbatucci [a], Jorge E. Fernández [a,b,*]

[a] Laboratory of Montecuccolino-DIN, Alma Mater Studiorum University of Bologna, via dei Colli 16, 40136 Bologna, Italy

[b] Alma Mater Studiorum Università di Bologna Representacion en la Republica Argentina, M. T. de Alvear 1149, Buenos Aires, Argentina

HIGHLIGHTS

- Pulse pile-up (PPU) is a common distortion in radiation detection.
- A balance equation for second order PPU is derived.
- A fast algorithm of the deterministic solution for rectangular pulses (DRPPU) is presented.
- The DRPPU algorithm is tested on experimental measurements obtained with a Si SSD.
- The DRPPU results are comparable with the ones obtained with a more sophisticated methodology.

ARTICLE INFO

Article history:
Received 26 October 2015
Received in revised form
6 September 2016
Accepted 10 September 2016

Keywords:
Pulse pile-up
Deterministic method
Deterministic solution
X-Ray spectrum

ABSTRACT

Pulse pile-up (PPU) is an always present effect which introduces a distortion into the spectrum measured with radiation detectors and that worsen with the increasing emission rate of the radiation source. It is fully ascribable to the pulse handling circuitry of the detector and it is not comprised in the detector response function which is well explained by a physical model. The PPU changes both the number and the height of the recorded pulses, which are related, respectively, with the number of detected particles and their energy. In the present work, it is derived a first principles balance equation for second order PPU to obtain a post-processing correction to apply to X-ray measurements. The balance equation is solved for the particular case of rectangular pulse shape using a deterministic iterative procedure for which it will be shown the convergence. The proposed method, deterministic rectangular PPU (DRPPU), requires minimum amount of information and, as example, it is applied to a solid state Si detector with active or off-line PPU suppression circuitry. A comparison shows that the results obtained with this fast and simple approach are comparable to those from the more sophisticated procedure using precise detector pulse shapes.

1. Introduction

Radiation emission is a process randomly spaced in time that, in a first approximation, can be considered as a Poisson process. During an X-ray measurement each photon absorbed in the detector produces a signal which is shaped by the pulse handling circuitry before arriving to the multi-channel analyzer (MCA). In this shaping process pulses with a too short time gap can be overlapped in a single pulse. This overlapping is called pulse pile-up and it is responsible of evident distortion in the measured spectra. PPU phenomena is common not only in classical solid state detectors (SSD), but also on the pixelated ones as reported in Rink et al., (2013) and Ballabriga et al. (2016). Usually for obtaining spectra with good statistics a high counting rate is recommended. The PPU effect increases with the counting rate because the pulse handling circuitry cannot process correctly too many pulses arriving in the same time interval.

Let us define the dead time τ of the counting system as the minimum amount of time necessary to record two pulses as separated events. If the time gap between two pulses is lower than the dead time, they cannot be distinguished and are recorded as a single pulse at a distorted energy (PPU effect). It is possible that not only two but also three or more pulses (a pulse train) are recorded as one. The number of photons piled-up together is called the order of the pile-up. PPU has three important consequences: in first place, the recorded spectrum suffer a net loss of counts; secondly, piled-up photons are assigned to wrong energy channels

* Corresponding author at: Laboratory of Montecuccolino-DIN, Alma Mater Studiorum University of Bologna, via dei Colli 16, 40136 Bologna, Italy.
E-mail address: jorge.fernandez@unibo.it (J.E. Fernández).

http://dx.doi.org/10.1016/j.radphyschem.2016.09.014
0969-806X/© 2016 Elsevier Ltd. All rights reserved.

Please cite this article as: Sabbatucci, L., Fernández, J.E., First principles pulse pile-up balance equation and fast deterministic solution. Radiat. Phys. Chem. (2016), http://dx.doi.org/10.1016/j.radphyschem.2016.09.014

and, finally, the whole spectrum is distorted since the lost pulses are not collected at the proper energies. Different post processing methods have been developed for characterizing the PPU and to obtain a corrected high counting spectrum. Analytical techniques based on first principles where implemented in the works of Wielopolski and Gardner (1976) and Taguchi et al. (2010).

Statistical methods were also implemented. The Monte Carlo Approach was introduced by Bristow and Harrison (1991). In order to make the Monte Carlo strategy adaptable to different kind of detectors Sabbatucci et al. (2014) proposed the code MCPPU which allows considering the specific pulse shape of the detector. Even if this last approach gives very accurate results, it is apparent the presence of numerical artifacts whose origin is not clear.

In the present work, it is described a post-processing analytical procedure which considers only second order PPU and a rectangular pulse shape (DRPPU). The aim of this work is to provide a simple and fast strategy for PPU correction which requires a minimum amount of information for being applied (the dead and live time and the measurement). The essential nature of this approach makes it possible to analyze the real origin of the even present numerical artifacts, free from Monte Carlo statistical errors. The solutions of both approaches will be compared for some paradigmatic examples. In particular, it will be considered a Si SSD with PPU suppression circuitry either activated or partially offline.

2. Second order PPU balance equation

Let us consider a measured spectrum $y(E)$ in a continuous energy range and the related measured counts in each infinitesimal energy bin $[E, E + dE]$, i.e. $y(E)dE$. From the physical point of view it is considered the measured spectrum before the action of the MCA. The $y(E)$ spectrum can be related to the original spectrum $h(E)$, free from PPU effect, through the following counts balance equation:

$$y(E)dE = h(E)dE - L(E)dE + R(E)dE, \tag{1}$$

where $L(E)dE$ and $R(E)dE$ denote, respectively, the probable number of lost and incoming pulses due to PPU in the generic bin $[E, E + dE]$. Let us introduce the normalized original spectrum $\bar{h}(E)$:

$$\bar{h}(E) = \frac{h(E)}{\int_0^\infty h(E')dE'} = \frac{h(E)}{N_t}, \tag{2}$$

and the related quantity $\bar{h}(E)dE$ which represents the probability of having one pulse with energy between E and $E + dE$. The quantity N_t represents the total number of pulses in the orginal spectrum. Assuming the generation of different pulses as an ensemble of completely independent processes the probability to have two pulses with energy in $[E_1, E_1 + dE_1]$ and in $[E_2, E_2 + dE_2]$ is simply $\bar{h}(E_1)\bar{h}(E_2)dE_1dE_2$. In order to find the mathematical expressions for both $L(E)dE$ and $R(E)dE$, let us define the differential PPU probability $\left(\partial\bar{P}/\partial E\right)dE$. This quantity represents the probability of having second order PPU with the resulting pulse energy lying in $[E, E + dE]$. The integral over all the possible resulting energies represents the total probability $\bar{P}$ of having second order PPU. Using the total PPU probability the expression of $L(E)dE$ is:

$$L(E)dE = 2\bar{P}h(E)dE, \tag{3}$$

The probability of having PPU, considering the pulse coming from $[E, E + dE]$ as the first or the second pulse, is the same and so the factor 2 is introduced.

Regarding $R(E)dE$ the probability of having two pulses, with energy in $[E_1, E_1 + dE_1]$ and $[E_2, E_2 + dE_2]$, which piled-up together produce a new pulse with energy in $[E, E + dE]$ is $\bar{h}(E_1)\bar{h}(E_2)(\partial\bar{P}/\partial E)dE_1dE_2dE$. By integrating this expression and multiplying it for N_t we obtain:

$$R(E)dE = N_t\left(\int_0^\infty\int_0^\infty \bar{h}(E_1)\bar{h}(E_2)\frac{\partial\bar{P}}{\partial E}dE_1dE_2\right)dE. \tag{4}$$

By replacing Eqs. (3) and (4) in Eq. (1) we obtain the nonlinear integral balance equation:

$$y(E) = (1 - 2\bar{P})h(E) + N_t\int_0^\infty\int_0^\infty \bar{h}(E_1)\bar{h}(E_2)\frac{\partial\bar{P}}{\partial E}dE_1dE_2. \tag{5}$$

3. PPU probabilities for a generic pulse shape

In order to find the mathematical expression of $\left(\partial\bar{P}/\partial E\right)dE$ it is possible to adapt, to our case, the method proposed by Wielopolski and Gardner (1976) for finding PPU probabilities. Assuming the generation of pulses as a Poisson's process, the probability that n uncorrelated events (generations) occur in the time interval, from 0 to t, is:

$$P_n(t) = \frac{(\lambda t)^n}{n!}\exp(-\lambda t), \tag{6}$$

where λ is the original counting rate not affected by PPU. Let us consider (see Fig. 1) two pulses, with energies E_1 and E_2, one generated at $t=0$ and the other generated in $t \in [t_1, t_1 + dt_1]$ with $t_1 < \tau$.

Following the Bayes theorem we can define the differential probability $d\bar{P}$ that two pulses are piled-up together as the product of three separated probabilities:

$$d\bar{P} = P_0(t_1)P_1(dt_1)P_0(\tau - t_1 - dt_1), \tag{7}$$

where using the Poisson formula we have:

$$P_0(t_1) = \exp(-\lambda t_1), \tag{8}$$

$$P_1(dt_1) = \lambda dt_1\exp(-\lambda dt_1) = \lambda dt_1, \tag{9}$$

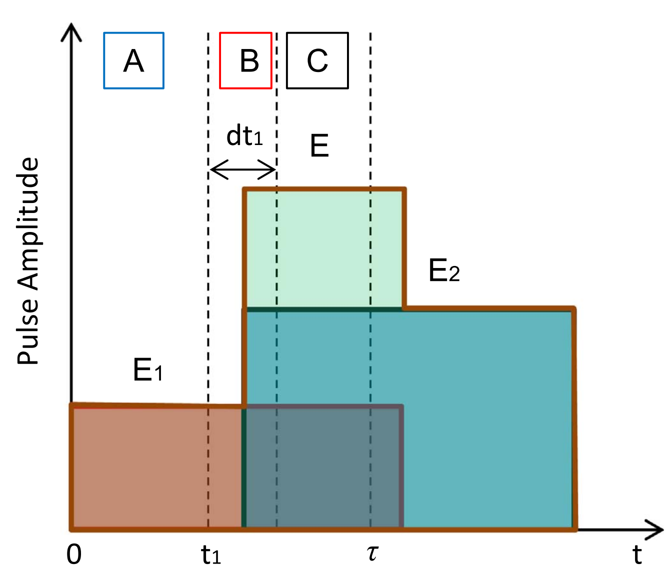

Fig. 1. PPU effect between two rectangular pulses with energy E_1 and E_2 which produce a new pulse with energy E. The second pulse is assumed to be generated in a time distance $[t_1, t_1 + dt_1]$ where t_1 is less than the dead time τ.

Please cite this article as: Sabbatucci, L., Fernández, J.E., First principles pulse pile-up balance equation and fast deterministic solution. Radiat. Phys. Chem. (2016), http://dx.doi.org/10.1016/j.radphyschem.2016.09.014