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Application of conservation theorem and modified extended tanh-function method to (1+1)-dimensional nonlinear coupled Klein–Gordon–Zakharov equation

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ABSTRACT

We found trivial conservation laws by conservation theorem and exact solutions modified extended tanh-function method of (1+1)-dimensional nonlinear coupled Klein–Gordon–Zakharov equation. The travelling wave solutions are expressed by the hyperbolic, trigonometric and rational functions. It is shown that the suggested method provides a powerful mathematical instrument for solving nonlinear equations in the science of mathematics, physics and engineering.

1. Introduction

Nonlinear partial differential equations (PDEs) have become useful for science and engineering. Nonlinear partial differential equations are not arised only from many fields of mathematics, but also from different discipline such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematic, chemical physics and geochemistry. One of the most powerful method for obtaining solutions of nonlinear partial differential equations is based on the study of their invariance under one parameter Lie group of point transformations. This method is used to analyze the symmetries of the differential equations. Then, obtained symmetry groups can be also used to simplify the given differential equations. It is known that conservation laws play an important role in the solutions of differential equations or system of differential equations. All of the conservation laws for PDEs have no physical meanings, but they are necessary for studying the integrability of PDEs [11]. At the mathematical level, conservation laws are highly connected to the existence of a variational principle which enables symmetry transformations. This crucial fact was fully acknowledged by Emmy Noether in 1918 [8]. Studies of consevation laws of PDEs are becoming attractive in nonlinear science day by day. Some matematicians such as Steudel, Anco and Bluman, Wolf, Kara and Mahomed, Ibragimov, Olver have numerous studies in that field [23]. There are lots of ways for construction of conservation laws. For example; characteristic method, variational approach [25], symmetry and conservation law relation [4,18], direct construction method for conservation laws [2], partial Noether approach [24], Noether approach [22], conservation theorem [5,12,14,15]. In the literature, it is mostly studied with partial differential equations containing real valued dependent variables to find conservation laws but in this study we will deal with partial differential equations containing complex valued dependent variables. Therefore we will consider (1+1)-dimensional nonlinear coupled Klein-Gordon-Zakharov equation. We will apply conservation theorem to this equation for finding it's conservation laws. This method is based on the concept of Lie symmetry generators, adjoint equations and formal Lagrangians.

We will deal with the exact solutions after finding the conservation laws of the given equation.

Exact traveling wave solutions of nonlinear PDEs are important for understanding qualitative property of many various areas of natural science. For inctance, exact solutions let researchers to design and operate tests by creating suitable conditions and determine these parameters or functions. Recently, many new methods have been proposed for finding these solutions, such as Jacobi elliptic function method [9], Weierstrass elliptic function method [7], Darboux and Backlund transform [21], symmetry reduction method [28], the tanh method [3], auxiliary method [16], extended tanh method [30], sine-cosine method [29], homogeneous balance method [10], exp-function method [34], the (G'/G)-expansion method [20], the modified simple equation method [17] first integral method [27], $\exp(-\phi(\xi))-$expansion method

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[19], extended-tanh method [37], modified extended tanh-function method [33–36].

In this paper, we will search conservation laws with conservation theorem and then we will obtain exact traveling wave solutions with modified extended tanh-function method of (1+1)-dimensional nonlinear coupled Klein–Gordon–Zakharov equation. In Section 2, we will define necessary preliminary informations about conservation theorem and modified extended tanh-function method. In Section 3, we will apply these methods to the (1+1)-dimensional nonlinear coupled Klein–Gordon–Zakharov equation. In Section 3.1, firstly, we will reduce given system to different form. Then, we will obtain Lie symmetry generators, formal Lagrangian, adjoint equations of founded system. Next, we will find trivial conservation laws for this system. At last, we will convert founded conservation laws for the given system. In Section 3.2, we will reduce given system to nonlinear ordinary differential equation. Then, we will find exact traveling wave solutions in the form of hyperbolic, trigonometric, rational finction solutions for (1+1)-dimensional nonlinear coupled Klein–Gordon–Zakharov equation. Finally, some conclusions will be given.

2. Preliminary informations

2.1. Conservation laws

Consider the kth order system of partial differential equations in the following form

$$E^\alpha\left(x, u, u_{(1)}, \dots, u_{(k)}\right) = 0, \quad \alpha = 1, \dots, m \tag{1}$$

here $x = (x_1, x_2, \dots, x_n)$ are n independent variables, $u = (u^1, u^2, \dots, u^m)$ are m dependent variables and $u_{(k)}$ denotes kth order partial derivatives, i.e., $u_i^\alpha = D_i(u^\alpha),\ u_{ij}^\alpha = D_jD_i(u^\alpha)$. D_i; means differentiation with respect to x_i, namely total differentiation operator. Total differentiation operator is shown that

$$D_i = \frac{\partial}{\partial x_i} + u_i^\alpha \frac{\partial}{\partial u^\alpha} + u_{ij}^\alpha \frac{\partial}{\partial u_j^\alpha} + \dots, \quad i = 1, \dots, n. \tag{2}$$

The infinitesimal generator for the governing Sys. (1) can be written as follow

$$X = \xi^i \frac{\partial}{\partial x_i} + \eta^\alpha \frac{\partial}{\partial u^\alpha}, \tag{3}$$

where ξ^i and η^α are called infinitesimal functions, $i = 1, 2, \dots, n$ and $\alpha = 1, 2, \dots, m$. The kth prolongation of the infinitesimal generator is

$$X^{(k)} = X + \eta_i^{(1)\alpha} \frac{\partial}{\partial u_i^\alpha} + \dots + \eta_{i_1 i_2 \dots i_k}^{(k)\alpha} \frac{\partial}{\partial u_{i_1 i_2 \dots i_k}^\alpha}, k \geq 1 \tag{4}$$

where

$$\begin{aligned} \eta_i^{(1)\alpha} &= D_i\eta^\alpha - \left(D_i\xi^j\right)u_j^\alpha \\ \eta_{i_1 i_2 \dots i_k}^{(k)\alpha} &= D_{i_k}\eta_{i_1 i_2 \dots i_{k-1}}^{(k-1)\alpha} - \left(D_{i_k}\xi^j\right)u_{i_1 i_2 \dots i_{k-1} j}^\alpha \end{aligned} \tag{5}$$

here $i, j = 1, 2, \dots, n$ and $\alpha = 1, 2, \dots, m$ and $i_l = 1, 2, \dots, m$ for $l = 1, 2, \dots, k$ and D_i is the total differentiation operator.

Sys. (1) has always formal Lagrangian by

$$L = w^\alpha E^\alpha \tag{6}$$

where $w^\alpha = w^\alpha(x_i, u^\alpha)$ new adjoint variables.

The Euler–Lagrange operator is given by

$$\frac{\delta}{\delta u^\alpha} = \frac{\partial}{\partial u^\alpha} + \sum_{s\geq 1}^{\infty} (-1)^s D_{i_1} \dots D_{i_s} \frac{\partial}{\partial u_{i_1 \dots i_s}^\alpha}, \quad \alpha = 1, \dots, m. \tag{7}$$

here $\frac{\delta}{\delta u^\alpha}$ is variational derivative. Then, adjoint equation system can be obtain with

$$(E^\alpha)^* = \frac{\delta L}{\delta u^\alpha}. \tag{8}$$

Theorem 1. *Every Lie point symmetry of Sys. (1) yields a conservation law for Sys. (1) and Sys. (8). Conserved vectors are described with following formulea*

$$T^i = \xi^i L + W^\alpha \frac{\delta L}{\delta u_i^\alpha} + \sum_{s\geq 1} D_{i_1} \dots D_{i_s}(W^\alpha) \frac{\delta L}{\delta u_{i i_1 \dots i_s}^\alpha} \tag{9}$$

where $i = 1, \dots, n,\ \ \alpha = 1, \dots, m$ *and* $W^\alpha = \eta^\alpha - \xi^j u_j^\alpha$. *The conserved vectors are obtained from which Eq. (9) involves the arbitrary solutions* w^α *of the adjoint equation and hence one obtains an infinite number of conservation laws for Sys. (1) by specifying* w^α.

Theorem 2. *The* $n-$*tuple vector* $T = (T^1, T^2, \dots, T^n)$*is a conserved vector of Sys. (1) if* T^i *satisfies*

$$D_i\left(T^i\right) = 0 \tag{10}$$

[26–32].

2.2. Modified extended tanh-function method

Consider the following nonlinear partial differential equation

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0 \tag{11}$$

where u is an unknown function and P is a polynomial of u and its partial derivatives, in which the highest order derivatives and the nonlinear terms are involved derivatives. We will tell steps of modified extended tanh-function method.

Step 1: We use the following wave transformation

$$u(x, t) = u(\xi),\ \ \xi = kx + wt \tag{12}$$

where k and w are constants. If we substitute wave transformations in the form of (12) in Eq. (11), then Eq. (11) reduces to following ordinary differential equation

$$F\left(u, u', u'', u''', \dots\right) = 0 \tag{13}$$

where the prime denotes the derivation with respect to ξ.

Step 2: Think that exact solution of Eq. (13) can be obtained in the following form

$$u(\xi) = a_0 + \sum_{i=1}^{n} \left(a_i(\phi(\xi))^i + b_i(\phi(\xi))^{-i}\right) \tag{14}$$

here a_i, b_i $(i = 0, 1, \dots, n)$ are constansts that will be determined later such that $a_n \neq 0$ or $b_n \neq 0$ and n is the balance term. We can determine positive integer balance term by balancing the highest order derivatives and the nonlinear terms. In Eq. (14), $\phi(\xi)$ satisfies the Riccati equation

$$\phi'(\xi) = b + (\phi(\xi))^2 \tag{15}$$

where b is a constant. Eq. (15) admits following solutions

Case 1: If $b < 0$, solution of Eq. (15) is

$$\phi(\xi) = -\sqrt{-b}\tanh\left(\sqrt{-b}\xi\right) \ \text{ or } \phi(\xi) = -\sqrt{-b}\coth\left(\sqrt{-b}\xi\right). \tag{16}$$

Case 2: If $b > 0$, solution of Eq. (15) is obtained by

$$\phi(\xi) = \sqrt{b}\tan\left(\sqrt{b}\xi\right) \text{ or } \phi(\xi) = \sqrt{-b}\cot\left(\sqrt{b}\xi\right). \tag{17}$$

Case 3: If $b = 0$, then

$$\phi(\xi) = -\frac{1}{\xi}. \tag{18}$$

Step 3: If we substitute Eq. (14) and its derivatives in Eq. (13) with the help of Eq. (15), we can get a polynomial in $\phi^i(\xi)$ $(i = 0, \pm 1, \pm 2, \dots)$. Setting all the coefficients of $\phi^i(\xi)$ equal to zero yields a set of over determined nonlinear algebraic equations for $a_0,\ k,\ w\ a_i,\ b_i$ $(i = 0, 1, \dots, n)$. When we solve obtained nonlinear algebraic equations using the Maple software, we can construct a variety of exact solutions for the nonlinear partial differential Eq. (11).