Chaos, Solitons and Fractals 104 (2017) 41–50

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Pinning synchronization of fractional order complex-variable dynamical networks with time-varying coupling

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ARTICLE INFO

Article history:
Received 21 December 2016
Revised 15 June 2017
Accepted 28 July 2017

Keywords:
Complex-variable dynamical networks
Caputo fractional order derivate
Time-varying coupling
Adaptive pinning control
Synchronization

ABSTRACT

This paper investigates the synchronization of fractional order complex-variable dynamical networks with time-varying coupling. Based on information of the complex network's configuration, an effective adaptive pinning control strategy to adjust simultaneously coupling strength and feedback gain is designed. Besides, we also consider the synchronization in complex networks with time-varying coupling weight. By constructing suitable Lyapunov function and using the presented lemma, some sufficient criteria are derived to achieve the synchronization of fractional order complex-variable dynamical networks under the corresponding update law. The update law is only dependent on the states of the complex dynamical networks, which do not need any other information such as the characteristic of the uncoupled nodes of the networks. Further, the result extends the synchronization condition of the real-variable dynamical networks to the complex-valued field. Finally, the correctness and feasibility of the proposed theoretical results are verified by two examples of fractional complex-variable dynamic networks.

1. Introduction

It is well known that complex network is a useful modeling tool for real-world system. Complex network refers to a network with self-organization, self-similarity, attractor, small world, and no scale [1]. This phenomenon is widespread in nature and society, such as the Internet, social networks, virus transmission network, food chain, power grids, and so on [2–4]. Because there are a large number of complex networks in our life, it is necessary to study the dynamical behavior of complex networks. In particular, as a typical collective behavior of dynamics, synchronization of the complex networks has attracted rapidly increasing attention in recent years due to its great applications in many real-world scenarios [5,6].

Synchronization is a phenomenon of network related behaviors, which is one of the most important phenomena in complex networks. Synchronization means that the state of all nodes in the network will eventually tend to be consistent with the evolution of time. It not only can explain a lot of natural phenomena, but also has potential applications in secure communication, image processing, engineering, etc. [7–9]. However, due to the complexity of the node dynamics and topology of the networks, all nodes are unable to achieve the synchronization themselves. Therefore, the proper controllers are required to achieve the objective, such as adaptive control [10], impulsive control [11], pinning control [12], intermittent control [13], etc. However, if we apply the controller to each node in the network, it is not realistic. Therefore, the researchers hope to find a more suitable control method.

Complex networks usually contain a large number of nodes and links. Due to the computational complexity and control cost, it is difficult to add controllers to every node in a very large-scale network. Reviewing the distributed nature of complex networks, it is reasonable to control them by acting locally on certain nodes and achieving synchronization of the whole network through coupling between the nodes, which is referred to as pinning control [14]. Pinning control is used to control the whole network by controlling a part of the nodes. Some of the existing works have shown that one can use the well-known pinning control method to synchronize the whole network [15–17]. This control method has greatly reduced the cost of control, so it has been used widely [18–20]. From the existing research results, according to the topology of complex networks, the controller can be applied to some of the key nodes in the network, which can achieve the purpose of network synchronization effectively. For example, in 2002, the pinning control strategy is introduced into the control of the scale free dynamic network, and it is effective to stabilize the scale of the chaotic dynamic network with large scale to the equilibrium point and get high control efficiency [21]. In [22], the authors

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http://dx.doi.org/10.1016/j.chaos.2017.07.028

proposed a general complex dynamical network model and then further investigated its adaptive pinning synchronization. Pinning and adaptive control measures are both low-cost and easy implement, which have been wide used in integer order complex dynamical networks.

When people study the complex system and complex phenomenon, especially the mechanical, biological and physical modeling problems, traditional integer order differential equation model has been unable to meet the people's needs, so the researchers expected to find a better mathematical modeling tool. Compared with the traditional integer order model, the fractional order model provides a good tool for the description of memory and hereditary properties of various substances and processes [23,24]. It would be more accurate if the practical problem is described by fractional order dynamic systems rather than integer order. Because of the fractional calculus can describe natural phenomena very well, it has a very wide range of applications in many fields, such as viscosity, biological engineering, dielectric polarization and so on [25–27]. At present, some new fractional order models have also been applied to the steady heat flow model [28–33]. In particular, the synchronization of fractional order complex dynamic networks start to attract more and more attention. Therefore, it is significant to study the fractional order complex networks.

However, previous researchers only considered the research of real-variable dynamic systems. In the real world, the systems can often evolve in different directions with a constant intersection angle with respect to complex number. From previous studies, the researchers found that complex-variable systems have a wide range of applications in various fields, such as the amplitude of electromagnetic field, the laser system and the thermal convection [34,35]. Since not only the complex-variable chaotic systems have great importance and broad applications, but also complex variables which double the number of variables can increase the content and security of the transmitted information. Therefore, in order to better describe the real world, a number of complex-variable dynamic systems have been proposed. For example, the complex Lorenz system was introduced to describe and simulate the rotating fluid and detuned laser [36]. A new hyperchaotic complex Lorenz system was generated from the complex Lorenz system and its dynamical characteristics were also analyzed and studied [37]. At the same time, an active control method was applied to realize phase and antiphase synchronization of two identical complex-variable hyperchaotic Lorenz systems [38]. In addition, Mahmoud constructed a new hyperchaotic complex Lorenz system [39], and studied adaptive anti-lag synchronization [40]. The stability of complex-variable impulsive system was solved in Ref. [41].

It should be noted that all of the above-mentioned works mainly focus on static networks, where the nodes and edges are constant. However, this assumption cannot be satisfied in many realistic situations, and the characteristics of many real-world networks are changing by different environmental conditions. In fact, in many real-world networks, the coupling strength is not a constant value and cannot be known in advance, but is automatically adjusted and varies with time according to different environmental conditions [42]. The effect of these uncertainties will destroy the synchronization and even break it. Therefore, the adaptive strategy has been presented to tune the coupling strength so as to guarantee synchronization in complex networks [43–45]. How to achieve their balance is challenging question. In this paper, the adaptive technique is used to solve this question. The coupling strength and feedback gains are tuned simultaneously to achieve their balance by designed adaptive laws. At present, the adaptive pinning synchronization of fractional order complex-variable dynamical networks is seldom studied and this fact motivates our work for the paper.

The main purpose of this paper is to study the synchronization of fractional order complex-variable dynamical networks with time-varying coupling. Up to now, although the synchronization of real-variable dynamical networks were investigated by many researchers, synchronization of dynamical networks with complex-variable systems is still an open problem that has not been studied widely. Therefore, this paper extends the synchronization results of the real-variable network to the complex-variable network. In actual networks, the consideration of coupling strength and feedback gain is very important. Generally, they are hoped to be as small as possible. To achieve synchronization, greater feedback gain is required if the coupling strength is too small, which increase the unnecessary cost. In this paper, the adaptive technique is used to solve this problem. The coupling strength and feedback gain are adjusted simultaneously by designing an adaptive law to achieve their balance. Therefore, we combine the adaptive strategy and the pinning control to design the effective controller, which is universal and reasonable for different dynamic networks. Meanwhile, the pinning and adaptive control schemes are both low-cost and easier to implement. In addition, if the size of the network is very large, the calculation of eigenvalues of the coupling matrix of the complex networks may be difficult to achieve. Therefore, we further investigate the synchronization of fractional order complex-variable dynamical networks with time-varying coupling weight. Under the proposed adaptive update law, the network can achieve synchronization without the need to calculate the eigenvalues of the outer coupling matrix. The update law is only dependent on the states of the coupled nodes of the complex dynamical network, which does not require any other information. Therefore, compared with the existing results, the synchronization method in this paper is more general and convenient to use. Finally, two numerical simulations are presented to demonstrate the effectiveness of the presented strategy.

The rest of the paper is organized as follows. In Section 2, some necessary preliminaries and the model of fractional order complex-variable dynamical networks are introduced briefly. Section 3 studies the network synchronization problem and derives some criteria. In Section 4, two numerical examples are provided to verify effectiveness of the proposed scheme. The conclusions are presented finally in Section 5.

Notations: Throughout this paper, for any Hermite matrix H, $H>0(H<0)$ means that the matrix H is positive (negative) definite. $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the corresponding matrix. For any complex number (or complex vector) x, x^r and x^i denote its real and imaginary parts, respectively. For any $x\in C^n$, $\bar{x}$ denotes the conjugate of x. x^H denotes conjugate transpose of x, $||x||=\sqrt{x^Hx}$ denotes the norm of x. $\otimes$ denotes the Kronecker product. $I_N\in R^{N\times N}$and $I_n\in R^{n\times n}$ are N-dimensional and n-dimensional identity matrix, respectively.

2. Preliminaries and network model

In this section, we give some preliminaries [46] and the basic definitions of fractional order complex-variable dynamic network which can be used to obtain our main results.

Definition 1. The fractional integral of order α for a function $f(x)$ is defined as

$$I_t^{\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)d\tau,$$

where $t\geq t_0$, $0<\alpha<1$, and $\Gamma(\cdot)$ is the Gamma function.