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Adaptive fuzzy impulsive synchronization of chaotic systems with random parameters

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ABSTRACT

Randomness is a common phenomenon in nonlinear systems. And conditions to reach synchronization are more complex and difficult when chaotic systems have random parameters. So in this paper, an adaptive scheme for synchronization of chaotic system with random parameters by using the fuzzy impulsive method and combining the properties of Wiener process and Ito differential is investigated. The main concepts of this paper are applying fuzzy method to approximate the nonlinear part of system, then using Ito differential to study the Wiener process of random parameters of chaotic system, and realizing synchronization under fuzzy impulsive method. The stability is analyzed by Lyapunov stability theorem. At the end of the paper, numerical simulation is presented to illustrate the effectiveness of the results obtained in this paper.

1. Introduction

Chaos control, stability analysis and synchronization are quite important topics in the field of nonlinear science since the pioneering research of Pecora and Carrols [1]. Thereinto, chaotic synchronization has become a hot topic in nonlinear systems because of its potential applications in secure communication, population dynamics, psychology, robotics and so on. Up to now, a great number of methods of control and synchronization for nonlinear chaotic systems have been proposed, such as sliding mode method [2,3], adaptive method [4,5], projection method [6], fuzzy method [7], impulsive method [8], and mixed control methods which mainly are adaptive fuzzy method [9], fuzzy impulsive methods [10,11], fuzzy sliding mode method [12] and so on.

These existing methods are usually devoted to achieving the synchronization of chaotic systems with certain parameters which were also called deterministic systems. However, uncertainties of parameters often appear in a chaotic system because of modeling error or measurement error. Generally speaking, the uncertainties of parameters is divided into uncertain parameters, unknown parameters and random parameters. Furthermore, some open studies show that these already existing methods of deterministic systems can be extended to solve nonlinear systems with uncertain even unknown parameters. Thus, some researches have realized synchronization of chaotic system with uncertainties [13–20]. For example, the synchronization of chaotic systems with uncertain parameters was realized based on adaptive method [13] and sliding mode method [14]. In paper [15], Yau et.al. developed a fuzzy sliding method to discuss the synchronization by combining fuzzy technology and sliding mode control method. And a robust adaptive fuzzy control method combining fuzzy technology and adaptive control method was proposed in [16]. A fuzzy impulsive control method based on fuzzy technology and impulsive method was developed in [17,18]. Some systems with uncertain parameters in special areas have also been studied, such as the modeling and adaptive back-stepping tracking control of neural network systems with uncertain parameters were studied by Jarina in [19], and the control of chaos in permanent magnet synchronous motors with uncertain parameters was discussed in [20]. In addition, discussions on the nonlinear systems with unknown parameters were also researched in paper [21–23].

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On the other hand, stochastic systems or the systems with random parameters have received much attention since stochastic modeling has come to play an important role in science and engineering applications. Aiming at random parameters or disturbances of chaotic systems or nonlinear systems, many research have proposed corresponding methods of synchronization [24–30]. In [24,25], a feedback method was applied to discuss the synchronization of two stochastic Duffing oscillators. The adaptive control and synchronization of chaotic systems with stochastic perturbation or stochastic parameters was discussed in [26]. In [27], adaptive Q-S method was developed to realize synchronization of coupled chaotic systems with stochastic perturbation and delay. Besides, Quasi-synchronization [28], exponential synchronization [29] and impulsive methods [30] were also used. In addition, synchronization of stochastic chaotic neural networks or chaotic neural networks with random parameters were well studied [31–40]. Exponential synchronization of chaotic neural networks with stochastic perturbations was discussed in [31,32]. Further, researches realized impulsive exponential synchronization of stochastic perturbed chaotic neural networks [33,34]. And then, other methods of synchronization for chaotic neural networks with stochastic perturbation were also studied, such as adaptive lag synchronization [35], intermittent control [36], adaptive synchronization [37,38], robust synchronization [39], anti-synchronization [40] and so on. Besides, some other theories like linear matrix inequalities [29], Ito differential [26] were applied to study the control and synchronization of stochastic systems or the systems with random parameters.

However, the nonlinear part of chaotic system with random parameters was usually eliminated directly or approximated roughly in the above research, which is not advocating. Even though fuzzy and impulsive method are methods which are easy to implement method, and successfully applied to solve synchronization of chaotic systems with deterministic parameters, uncertain parameters and unknown parameters, no researches tried to use these methods to realize synchronization of chaotic system with random parameters. To solve this problem, this paper investigates a new method to realize synchronization of chaotic system with random parameters. The method is combining adaptive, fuzzy and impulsive methods, using Wiener process to show the randomness and applying Ito differential to study Wiener process. Of course, the proposed adaptive fuzzy impulsive method is specifically designed for the study of stochastic problems. The precondition of this method is that randomness satisfies Wiener process. The main concepts of this paper are applying fuzzy method to approximate the nonlinear part, then using Ito differential to study the Wiener process of random parameters of chaotic system, and realizing synchronization under adaptive fuzzy impulsive method. The proposed adaptive fuzzy impulsive method simplifies the sufficient conditions for synchronization, and let the controller be easier to design. Although the proof is quite complex which needs the properties of Wiener process and Ito differential. In addition, it also enriches the method to realize synchronization of stochastic chaotic system or chaotic system with random parameters.

The rest of this paper is organized as follows. In Section 2, the fuzzy impulsive model of chaotic system with stochastic parameters is established. In Section 3, the controller is designed and the synchronization of chaotic systems with random parameters is analyzed based on the Lyapunov Stability Theorem and the Ito differential. Numerical simulations are given to verify our theoretical results in Section 4. Concluding remarks is drawn in Section 5.

2. Fuzzy impulsive model of chaotic system with random parameters

The mathematical model of a chaotic system can usually be written as following which is also called as matrix form of the systems,

$$\dot{x}(t) = Ax(t) + f(x(t)), \tag{1}$$

where $x(t) \in R^n$ represents the state variables of the chaotic system, $A \in R^{n\times n}$ and $f(x(t))$ is a continuous function with respect to its arguments.

Then, the chaotic system with random parameters can be characterized by

$$\dot{x}(t) = f(x(t)) + F(x(t))\xi, \tag{2}$$

where $\xi = (\xi_1, \xi_2, \dots, \xi_m)^T \in R^m$ denotes the random parameters of the chaotic system, and $F(x(t)) \in R^{n\times m}$.

Obviously, combining Eq. (1) with Eq. (2), we have $Ax(t) = F(x(t))\xi$. Besides, we assume that the value of parameters deviates randomly around their unknown average value ξ which satisfies

$$\xi = \bar{\xi} + \Phi\dot{W}, \tag{3}$$

where $W \in R^m$ is the vector obeying the Wiener process, $\bar{\xi}$ is the mean value of ξ, and matrix $\Phi \in R^{m\times m}$ is a known time-varying or state dependent matrix as well as with norm bounded ($\|\Phi\| \le M_\Phi$), and M_Φ is a known positive constant.

The T-S fuzzy models are usually described by fuzzy "IF-THEN" rules, in which each rule locally represents a linear input-output realization of the system over a certain region of the state space. Specifically, a general T-S fuzzy system of chaotic system is described as follows [30]:

Plant Rule i: if $x_1(t)$ is M_{i1}, $x_2(t)$ is M_{i2}, ..., $x_n(t)$ is M_{in}. Then

$$\dot{x}(t) = A_i x(t), i = 1, 2, \dots, r, \tag{4}$$

where M_i is the fuzzy set, r is the numbers of "if-then" rules, and $A_i \in R^{n\times n}$ which can be written as $A_i = A + C_i$, where $C_i \in R^{n\times n}$.

Then combining random parameters (3) and the fuzzy rules (4), chaotic system with random parameters (2) under ith rule can be rewritten as

$$\dot{x}(t) = C_i x(t) + F(x(t))\bar{\xi} + F(x(t))\Phi\dot{W}, i = 1, 2, \dots, r. \tag{5}$$

Adding the impulsive control to system (5), we get the impulsive chaotic system with random parameters under fuzzy ith rule as

$$\begin{aligned} \dot{x}(t) &= C_i x(t) + F(x(t))\bar{\xi} + F(x(t))\Phi\dot{W}, t \neq t_k \\ \Delta x(t_k) &= H_k x(t_k), k = 1, 2, \dots, \end{aligned} \tag{6}$$

where $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, and H_k is the impulsive matrix, a sequence of instants $\{t_k\}$ satisfies $0 < t_1 < t_2 < \dots$, and $\lim_{k\to\infty} t_k = +\infty$.

Let fuzzy impulsive model (6) be the drive system. Considering another chaotic system (which has different structure) with random parameters as the response system

$$\dot{y}(t) = By(t) + g(y(t)) + u(t), \tag{7}$$

where $y(t) \in R^{n\times n}$ represents the state variables, $B \in R^{n\times n}$, and $g(y(t))$ is a continuous function with respect to its arguments. $u(t)$ is the controller need to design.

Then the response system (7) can be written as the form of (2)

$$y(t) = G(y)\eta + g(y(t)) + u(t), \tag{8}$$

where $\eta = (\eta_1, \eta_2, \dots, \eta_l)^T$ denotes the random parameters of the chaotic system, $G(y) \in R^{n\times l}$, and the random parameters also satisfies

$$\eta = \bar{\eta} + \Psi\dot{U}, \tag{9}$$