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Asymmetric multifractal cross-correlations and time varying features between Latin-American stock market indices and crude oil market

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ABSTRACT

We apply MF-ADCCA to analyze the presence and asymmetry of the cross-correlations between Latin-American and US stock market indices and crude oil market. We find that multifractality exists in this cross-correlations, and that there is asymmetry on its behavior. The asymmetry degree changes accordingly to the series considered for the trend behavior. We find that fluctuation sizes greatly influence the asymmetry in the cross-correlation exponent, increasing for large fluctuations when we consider the trend of the crude oil price. We also find no clear differences in the exponents with different scales under different trends of the WTI, contrary to other studies in asymmetric scaling behavior. When we examine the time varying features of the asymmetry degree we find that the US indices show a consistent behavior in time for both trends, where the cross-correlation exponents tend to be larger for downward trends. On the other hand, given the more heterogeneous individual properties of Latin-American indices, the asymmetry behavior varies more depending on the trend considered.

1. Introduction

Despite the growth of alternative energy sources, oil still is and will be a key resource for the industrialized nations, playing an important role in almost all productive activities. For this reason, industrialized countries have felt the big impact of the oil price variations in the last decades, giving rise to a great number of investigations of the quantitative and qualitative effects of this variations in economic growth, exchange rates and stock markets, making essential the understanding of the dynamics in the cross-correlations.

Since the works by Hurst [1,2] and Mandelbrot [3] were published, a lot of work has been done identifying the power-law characteristics in varied real world series with the use of the Rescaled Range Analysis (R/S), such as mining [4], traffic flow [5], air pollutants [6], and economics [7–9]. However, this method presented sensitivity to the short term auto-correlation and the non-stationarities, which can lead to a biased estimation of long memory parameters [10]. With the publication of Peters' Fractal Market Hypothesis [11], a lot of work build on this hypothesis and tried to address the issues of the R/S, such as the Wavelet Transform Modulus Maxima (WTMM) method [12], the Fluctuation Analysis (FA) [13], the Detrended Fluctuation Analysis (DFA) [14], the Detrended Moving Average analysis (DMA) [15], its Multifractal extention (MF-DMA) [16] and the Multifractal Detrended Fluctuation Analysis (MF-DFA) [17] which has been extensively used in the analysis of financial series [18–23], and has given a new way to investigate market efficiency [23,24].

With the identification of the cross-correlations in many financial time series [25–32], other methods were developed, such as the Detrended Cross-Correlation Analysis (DCCA) [33], the Multifractal Detrending Moving Average Cross-Correlation Analysis (MF-X-DMA) [34], and the Multifractal Detrended Cross-Correlation Analysis [35] (MF-DCCA) which integrates DCCA into the MF-DFA framework. Since then, the MF-DCCA method has been widely used in analysis of financial series [36–41]. However, the methods mentioned above do not consider the asymmetric market responses to different economic news [42–45]. In this study we are interested in studying the cross-correlations under different trends of one market. To this end, we apply the Multifractal Asymmetric Detrended Cross-Correlation Analysis method (hereinafter MF-ADCCA) proposed by Cao et al. [46]. This method is a combination of the MF-DCCA method, and the asymmetric version of the DFA, the A-DFA proposed by Alvarez-Ramirez et al. [47]. It has revealed interesting features of stock markets [50], commodities [49] and energy markets [48]. By using this method, we can study the multifractal properties in cross-correlations without falling in the subjective sample division between bull and bear markets.

This study focuses on six Latin-American stock market indices, on which the major portion of the Latin-American economy de-

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pends, and are also the major recipients of foreign investment in the last decades. Of the six countries, Mexico, Brazil, Argentina and Colombia are oil producers, making Latin-America a net exporter in the global oil market. On the other hand, Chile and Peru are net importers, so countries with different relationships with the oil market are studied. Understanding the dynamics of this relationship is pivotal for the government, multinational organizations and investors in this region to prepare for potencial adverse movements in the economy.

To the best of our knowledge, this is the first study of the multifractal and asymmetric properties of the cross-correlations between the crude oil price and the Latin-American stock market indices. We also incorporate the rolling window method to investigate the time varying dynamics of the asymmetry degree.

The remaining of this paper is organized as follows. Section 2 describes the method used, Section 3 describes the data used in the analysis. Section 4 provides the results obtained. Finally Section 5 concludes the paper.

2. Multifractal asymmetric detrended cross-correlation analysis method

The MF-ADCCA [46] is based on the A-DFA and the MF-DCCA, and it allows to examine the asymmetric multifractal characteristics of two cross-correlated time series. The method achieves great performance even with highly non-stationary series, and its steps are similar to the steps in the MF-DCCA method.

Assuming two time series x_i and y_i, $i = 1, \dots, N$, where N is the length of the series, the method can be summarized as follows.

- **Step 1**: Construct the profile

$$X(i) = \sum_{t=1}^{i}(x_t - \bar{x}), \quad Y(i) = \sum_{t=1}^{i}(y_t - \bar{y}), \quad i = 1, \dots, N \tag{1}$$

  Where $\bar{x}$ and $\bar{y}$ represent the average of the series in the whole period.
- **Step 2**: Divide the profiles $X(i)$ and $Y(i)$ into $N_s \equiv [N/s]$ non-overlapping windows of equal length s. Since the length of the series N is not necessarily a multiple of the time scale s, some part of the profile can remain at the end. In order to not discard this part, the same procedure is applied starting from the end of the series. This means that we obtain $2N_s$ segments.
- **Step 3**: The trends, $X^v(i)$ and $Y^v(i)$ for each one of the $2N_s$ segments are estimated by means of a linear regression as $X^v(i) = a_{X^v} + b_{X^v} \cdot i$ and $Y^v(i) = a_{Y^v} + b_{Y^v} \cdot i$. This precedes the determination of the detrended covariance, calculated as follows

$$F(v,s) = \frac{1}{s}\sum_{i=1}^{s}|X[(v-1)s+i] - X^v(i)| \cdot |Y[(v-1)s+i] - Y^v(i)| \tag{2}$$

  for each segment v, $v = 1, \dots, N_s$ and

$$F(v,s) = \frac{1}{s}\sum_{i=1}^{s}|X[N-(v-N_s)s+i] - X^v(i)| \cdot |Y[N-(v-N_s)s+i] - Y^v(i)| \tag{3}$$

  for each segment v, $v = N_s + 1, \dots, 2N_s$.
- **Step 4**: By means of averaging over all segments, the qth order of fluctuation function can be obtained as follows for the different behavior of the trends in time series x_t

$$F_q^+(s) = \left(\frac{1}{M^+}\sum_{v=1}^{2N_s}\frac{sign(b_{X^v})+1}{2}[F(v,s)]^{q/2}\right)^{1/q} \tag{4}$$

$$F_q^-(s) = \left(\frac{1}{M^-}\sum_{v=1}^{2N_s}\frac{-[sign(b_{X^v})-1]}{2}[F(v,s)]^{q/2}\right)^{1/q} \tag{5}$$

  when $q \neq 0$, and

$$F_0^+(s) = exp\left(\frac{1}{2M^+}\sum_{v=1}^{2N_s}\frac{sign(b_{X^v})+1}{2}[F(v,s)]^{q/2}\right)^{1/q} \tag{6}$$

$$F_0^-(s) = exp\left(\frac{1}{2M^-}\sum_{v=1}^{2N_s}\frac{-[sign(b_{X^v})-1]}{2}[F(v,s)]^{q/2}\right)^{1/q} \tag{7}$$

  for $q = 0$. $M^+ = \sum_{v=1}^{2N_s}\frac{sign(b_{X^v})+1}{2}$ and $M^- = \sum_{v=1}^{2N_s}\frac{-[sign(b_{X^v})-1]}{2}$ are the number of subtime series with positive and negative trends. We assume $b_{X^v} \neq 0$ for all $v = 1, \dots, 2N_s$, such that $M^+ + M^- = 2N_s$.
  The traditional MF-DCCA is implemented by computing the average fluctuation function for $q \neq 0$

$$F_q(s) = \left(\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F(v,s)]^{q/2}\right)^{1/q} \tag{8}$$

  and as follows when $q = 0$

$$F_q(s) = exp\left(\frac{1}{4N_s}\sum_{v=1}^{2N_s}ln[F(v,s)]\right) \tag{9}$$

- **Step 5**: The scaling behavior of the fluctuations is analyzed by observing the log-log plots of $F_q(s)$ versus s for each value of q. In the case where the two series are long-range cross-correlated, $F_q(s)$ will increase for large values of s, as a power law

$$F_q(s) \sim s^{H_{xy}(q)} \tag{10}$$

$$F_q^+(s) \sim s^{H_{xy}^+(q)} \tag{11}$$

$$F_q^-(s) \sim s^{H_{xy}^-(q)} \tag{12}$$

The scaling exponent $H_{xy}(q)$ is known as the generalized cross-correlation exponent, and describes the power-law relationship between two series. It can be obtained by calculating the slope of the log-log plots of $F_q(s)$ versus s through the method of Ordinary Least Squares (OLS).

In the case of $q = 2$, the generalized cross-correlation exponent has similar properties and interpretation as the univariate Hurst exponent calculated by the DFA. If $H_{xy}(2) > 0.5$, the series are cross-persistent, so a positive (negative) change in one price is more statistically probable to be followed by a positive (negative) value of the other price. In the case where $H_{xy}(2) < 0.5$ the series are cross-antipersistent, which means that a positive (negative) change in one price is more statistically probable to be followed by a negative (positive) change on the other price. For $H_{xy}(2) = 0.5$ only short-range cross-correlations (or no correlations at all) are present in the relationship between the series.

To measure the asymmetric degree of the cross-correlations we can calculate, for every q, the following metric

$$\Delta H_{xy}(q) = H_{xy}^+(q) - H_{xy}^-(q) \tag{13}$$

The greater the absolute value, the greater the asymmetric behavior. If $\Delta H_{xy}(q)$ is equal or close to zero, then the cross-correlations are symmetric for different trends of time series x_t. If the value of $\Delta H_{xy}(q)$ is positive, it means that the cross-correlation exponent is higher when the time series x_t has a positive trend that when it is negative. If it is negative, the cross-correlation exponent is lower