Chaos, Solitons and Fractals 104 (2017) 173–181

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Investigation and quantification of Phase coherence index for different types of forcing in DC glow discharge plasma

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ARTICLE INFO

Article history:
Received 22 December 2016
Revised 17 August 2017
Accepted 18 August 2017

Keywords:
Forcing
Recurrence plot
Structure function
Surrogate data
Phase coherence index
Continuous wavelet transform
Delay vector variance

ABSTRACT

The evidence of finite nonlinear interaction in a DC glow discharge plasma has been demonstrated by estimating phase coherence index for different types external forcing techniques likewise noise, sinusoidal, square etc. The existence of finite phase coherence index i.e finite correlation prompts us to carry out nonlinearity analysis using delay vector variance (DVV). Finite nonlinear interaction obtained from phase coherence index values is observed to be predominant at a particular amplitude of square forcing which corroborates our nonlinearity analysis using DVV. Existence of phase coherence index has been demonstrated introducing continuous wavelet transform (CWT). Characterization of the difference in the phase distribution by the difference in the waveform in real space instead of dealing in Fourier space has been facilitated by introducing structure function or path length for different orders to study and identify the dynamical system. The expression of path length eventually enables us to evaluate the phase coherence index. The transition in the dynamics is observed through recurrence plot techniques which is an efficient method to observe the critical regime transitions in dynamics.

1. Introduction

The theory of nonlinear, deterministic dynamical systems provides powerful theoretical tools to characterize geometrical and dynamical properties of the attractors of such systems [1]. Substantial work has been carried out to understand and reveal different nonlinear processes and the hidden dynamics in order to gain relevant information about the system of interest. Numerous scientific disciplines such as astrophysics, biology, geoscience uses data analysis techniques to get an insight into the complex systems observed in nature [2] which show generally a non-stationary and complex behaviour although almost all methods of time series analysis [12], traditional linear or nonlinear, must assume some kind of stationarity [3]. As the complex systems are characterized by different transitions between regular, laminar and chaotic behaviors, the knowledge of these transitions is essential to understand underlying mechanism behind a complex system. Linear approaches are insufficient to study the changes in the dynamics during the measurement period that usually constitute an undesired complication of the analysis. Nonlinear approach such as recurrence plot analysis will be suitable to graphically detect different patterns and structural changes in time series data which exhibit characteristic large and small scale patterns caused by the typical dynamical behaviour [13]. In this report an attempt has been made to understand the hidden dynamics by adopting recurrence plot (RP) [27], along with structure function or path length which turns out to be a powerful tool in analysing a nonlinear, complex signal.

Plasma effects are finding ever increasing applications in astrophysics, solid state physics, physics of gas discharge and thermonuclear fusion [14]. Plasmas are intrinsically nonlinear whose effects manifest in the form of various exotic structures such as double layers [15], solitons, vortices, different types of waves, instabilities and turbulence [18]. Glow discharge plasma being rich in high energy, electrons and ions are capable of exhibiting many such nonlinear phenomenon [16,17]. So much effort has been endeavoured to study the intricacies involving the topics like finite nonlinear interactions and its associated phase coherence index. The investigation of the nonlinear wave interaction is based on the decomposition of a signal into its amplitude and phase part albeit we have to assume implicitly weak nonlinearity. From this point of view the amplitude along with the phase information obtained from the Fourier transform is convenient for our analysis permitting us wave number/frequency decomposition. Structure function or path length analysis [6–8] bears a significant aspect in this regard. Depiction of structure function has been executed for original (ORG) as well as for phase randomised (PRS), phase constant surrogate data (PCS) requiring detailed knowledge about the generation of surrogate time series. Quantification measure of the structure

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http://dx.doi.org/10.1016/j.chaos.2017.08.016
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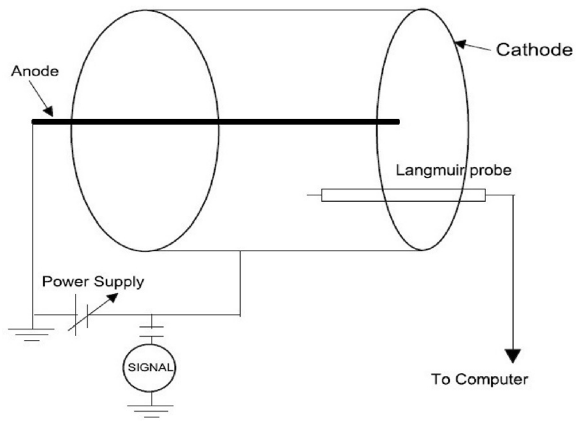

Fig. 1. Experimental setup for glow discharge plasma.

function has been adopted by estimating phase coherence index [6,9]. In fact the purpose of this paper is to discuss the nonlinear interaction and matters pertaining to phase information [4], nonlinearity [10,11] etc with emphasis on the calculation of the phase coherence index. This is the first time that the distribution of wave phases for two different kinds of surrogate data namely phase randomised, phase constant along with the original one has been executed by applying some external means of forcing. In order to avoid the influence of the choice of origin we make two surrogate data sets from the original FPF and characterise difference in real space by evaluating the path length or equivalently structure function. The nonlinear interaction not only contributes to the exchange of energy among the wave modes [4], but also to the synchronisation of the wave phases.

The paper is organized as follows. In Section 2, we present a brief schematic of the experimental setup, followed by the results of the analysis of recurrence plot, structure function, phase coherence index respectively in Sections 3 and 4. Section 5 represent a comprehensive analysis of nonlinearity with surrogate data as well as with delay vector variance method. Conclusions are presented in Section 6.

2. Experimental setup

The experiments were carried out in a cylindrical hollow cathode DC glow discharge argon plasma with a typical density and temperature of $\sim 10^7/\text{cm}^3$ and 2–6 eV respectively. The chamber was evacuated by rotary pump to attain a base pressure of 0.001 mbar. Experiments were performed under different forcing condition like sinusoidal, square, noise keeping operating neutral pressure fixed at 0.12 mbar with discharge voltage (DV) being fixed at 435 V. An unbiased Langmuir probe was used to obtain the floating potential fluctuations acquired with a sampling time of 10^{-6} s. A signal generator was coupled with the DV through a capacitor for observing fluctuations in presence of forcing as shown in the schematic diagram of Fig. 1.

3. Floating potential fluctuation, recurrence plot

The sequential change in floating potential fluctuations (FPF) acquired by applying noise, sinusoidal and square forcing is presented in Fig. 2. The use of three external forcing, i.e. sinusoidal, square and noise is very important from the practical point of view in any forced dynamical system. Phenomena like phase locking, which is very important in the chaos control can be achieved using the sinusoidal forcing. Similarly, other phenomenon like period pulling, resonance, etc. require the use of sinusoidal and square forcing. As far as forcing is concerned noise is omnipresent in any experimental system. Also, the phenomenon like stochastic resonance (enhancement of sub- threshold signal in the presence of external noise forcing) and coherence resonance can be achieved using noise forcing. Due to all these physical applications, we tried to explore the nonlinearity and other feature of plasma dynamics in the presence of three different forcing. The dynamical change in the FPF's has been detected with recurrence plots (RP) and discussed in the framework of RP.

The recurrence plot (RP) is a relatively new technique of time series signals to understand the hidden insights involving the intricacies of the interplay between nature of different periodicity of the system and was introduced by Eckman and Kamphorst [19]. The RP expressed as a two dimensional square matrix (in Eq. (1)) represents the occurrence with ones and zeroes for states X_i and X_j and find the hidden periodicity in a time series signal which is not observable by naked eye.

$$R_{ij} = H(\epsilon - ||X_i - X_j||); \mathbf{i}, \mathbf{j} = \mathbf{1}, \ldots, \mathbf{M} \tag{1}$$

where $\mathbf{M}$ is the number of data points of the signal, H is the heaviside function and $||.||$ is the norm (Euclidean norm), ϵ is the choice of the threshold. A crucial parameter of RP is the threshold ϵ. Therefore, special attention has to be required for its choice. If ϵ is chosen too small, there may be almost no recurrence points and we cannot learn anything about the recurrence structure of the underlying system. On the other hand, if ϵ is chosen too large, almost every point is a neighbour of every other point, which leads to a lot of artefacts. For this case it was proposed to choose such that the recurrence point density is selected to be approximately 1% [20] in our case.

According to Takens embedding theorem, for a time series data X_i, an embedding can be made using the vector $Y_i = X_i, X_{i+\tau} \ldots X_{i+(d-1)\tau}$ which represent the original time series embedded into d dimensional phase space with τ being the delay. RP's are graphical, two dimensional representations showing the instants of time at which a phase space trajectory returns approximately to the same regions of phase space. A recurrence is said to occur whenever a trajectory visits approximately the same region of phase space indicating $R_{ij} = 1$, whereas if the state does not recur with itself we are left with $R_{ij} = 0$.

RP's of FPF's are depicted in Fig. 3 for different types of external perturbation for the qualitative analysis and visualisation of the recurrences of dynamical system. Starting with the Fig. 3a and b (corresponding to the time series fluctuation of first and last subplot in the left panel of Fig. 2a) without any external forcing and for the noise forcing amplitude of **A=4 V** we can hardly observe any point indicating almost zero recurrence followed by the appearance of distinct diagonal lines with scattered points in between long diagonal lines for square forcing of amplitude of **A=1 V**. The arrangement of the scattered point for square forcing of **A=2 V** (Fig. 3d) occupies a large region in between the bold diagonal lines in recurrence plot with increasing number of diagonal lines. The RP's in Fig. 3e shows a long diagonal line with some faint signature of non-diagonal lines which becomes more ordered and prominent for increasing value of sinusoidal forcing depicted in Fig. 3f indicating deterministic behaviour. We have clearly delineated the ordered behaviour in Fig. 4 through RP plots in presence of increasing sinusoidal forcing. Initially in Fig. 4b we are left with a main diagonal line with some other faint diagonal lines. The arrangement of the broken diagonal lines along with the scattered points within main long diagonal lines are seen to become more ordered in Fig. 4c and d at forcing amplitudes of A=2, 2.4 V. At higher forcing amplitudes of 5 V, 6 V, RP plots exhibit some prominent arrangement of long diagonal lines. Diagonal lines in the plots are indicative of periodic, deterministic behaviour and represent similar evolution of states at different times.