Chaos, Solitons and Fractals 104 (2017) 389–399

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Effects of toxicants on Phytoplankton-Zooplankton-Fish dynamics and harvesting

Prabir Panja [a,*], Shyamal Kumar Mondal [a], Dipak Kumar Jana [b]

[a] Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore -721102, W.B., India
[b] Department of Applied Science, Haldia Institute of Technology, Haldia, Purba Medinipur-721657, W.B., India

ARTICLE INFO

Article history:
Received 21 May 2017
Revised 14 August 2017
Accepted 24 August 2017

Keywords:
Phytoplankton
Zooplankton
Fish
Hopf bifurcation
Pontryagin's maximum principle
Harvesting

ABSTRACT

In this paper, a three species predator-prey interaction model among Phytoplankton, Zooplankton and Fish has been developed in the presence of toxicant. It is assumed that Phytoplankton grows logistically and its growth rate is affected by toxin release in the environment through different natural and human activities. It is considered that Zooplankton consumes only Phytoplankton. Fish consumes both Phytoplankton as well as Zooplankton. Here, it is also assume that Phytoplankton releases some toxin in the environment which makes some death on Zooplankton. Also, it is considered that toxicant in the environment are increased constantly for different natural and human behavior. Then, different equilibrium points have been determined and the stability of the proposed system has been analyzed around these equilibrium points. Hopf bifurcation analysis has been done with respect to the contact rate between Phytoplankton and Environmental toxin (γ), constant rate of increase of Environmental toxin (A) , depletion rate of environmental toxin (d_1), harvesting effort (E) and rate of releasing toxin by Phytoplankton (ρ). From this study, it is seen that these parameters have a big role on the stability of this predator prey system. Finally, some numerical simulation results have been shown to verify our analytical findings.

1. Introduction

Mathematical Biology is the application of mathematical modelling to solve problems in biology and physiology. It is one of the fastest growing research areas in mathematics and is contributing significantly to our understanding of the biological world and the processes in disease. Phytoplankton are autotrophic component of the plankton community and they are the primary producer of Oceans, Seas and Fresh water ecosystem. A variety of toxins can be produced by Phytoplankton. Some species of the Phytoplankton which produces toxin are Alexandrium sp., Amphidinium carterae, Chrysochromulina polylepis, Cooliamonotis, Dinophysis spKeeping etc. Zooplankton are tiny animal in the water bodies, they consume Phytoplankton. Since in a food chain Phytoplankton acts as a primary producer, so the research on Phytoplankton, Zooplankton and Fish dynamics is very much essential for our society due to their universal existence and importance.

There exist many mathematical models describing the dynamics of Phytoplankton and Zooplankton interaction. In 2003, Sarkar and Chattopadhyay [22] studied a mathematical model on the occurrence of planktonic blooms under environmental fluctuations and the possible control of blooms through experimental observations. From this study, they observed that toxin producing Phytoplankton (TPP) and control of the rapidity of environmental fluctuation are key factors for the termination of planktonic blooms. Das and Ray [23] investigated a Phtoplankton-Zooplankton system with nutrient as a delay parameter. They showed that delay in the decomposition process of dead Phytoplankton from senescence and other mortalities do not affect the stability of the system. After that Lv et al. [24] studied a Phytoplankton-Zooplankton interaction mathematical model in the presence of harvesting on both the population. From this study, they concluded that over exploitation can result the extinction of a population and for the existence of a population optimal use of harvesting is very much essential. Also, it is known to all that Fish is a good source of food for the human beings along the whole world. Fish consumes both Phytoplankton as well as Zooplankton. So, the study on the dynamics of Phytoplankton, Zooplankton and Fish is very necessary. In 2015, Panja and Mondal [4] investigated a predator prey interaction model among Phytoplankton, Zooplankton and Fish. This study ensures that the conservation rate of Phytoplankton by Zooplankton and rate of toxin release by Phytoplankton has an important role on the stability of the system. The role of toxin and nutrient for the occurrence and termination of planktonic bloom has been studied by Pal et al.

* Corresponding author:
E-mail addresses: prabirpanja@gmail.com (P. Panja), shyamal_260180@yahoo.com (S.K. Mondal), dipakjana@hithaldia.ac.in (D.K. Jana).

http://dx.doi.org/10.1016/j.chaos.2017.08.036
0960-0779/

[5]. Again, Chattopadhyay and Pal [6] investigated the impact of viral infection on the stability of Phytoplankton-Zooplankton system mathematically. Also, Chattopadhyay et al. [7] studied that the toxin producing Phytoplankton may control the planktonic blooms through field study and mathematical modelling.

Due to different human activities such as rapid industrialization, extreme use of pesticide in agricultural system, modern lifestyle etc. makes our ecological system polluted day by day. Phytoplankton, Zooplankton, Fish and other aquatic organism have been greatly affected by toxins. In 1980, Moratou-Apostolopoulou and Ignatiades [2] investigated that the pollution effects on the growth of Phytoplankton and the interaction dynamics among Phytoplankton and Zooplankton. Hallam et al. [8–10] studied the effects of toxicants on population and from their study, they showed that the toxicant-population can lead to multiple stable equilibria where in the absence of the toxicant, the population has only a single stable equilibrium. In 1996, Chattopadhyay [15] studied the effect of toxic substances on a two-species competitive system. In this paper, it is shown that toxic substance has an important effects on the stability of the predator prey system. Tchounwou et al. [1] published an article on the effects of heavy metals toxicity on the Environment. They suggested that metals toxicity have a great impact on the living organism on the water bodies. Jun Huang et al. [3] showed that oil pollution on marine Phytoplankton has the chronic effects in a subtropical bay, China. There are many studies [11,13,14,16–20] theoretically and experimentally proved the impact of toxicant on living animals in an aquatic system. So, these studies influence us to study the dynamics of Phytoplankton, Zooplankton and Fish system in the presence of toxicants.

Again, harvesting is the process of collecting a ripe crop from the fields. Harvesting is necessary for the economic development of a country. Again, many people throughout the whole world are dependent on the harvesting of Fish, since it is a favorable food source for human beings. But continuous or over harvesting on a particular species may cause the extinction of that species. So, a suitable optimal control strategy is necessary for the conservation of species. In 2012, Chakraborty et al. [12] studied a stage structure predator prey model and the harvesting on the predator population. After that Gupta and Chandra [27] investigated a modified Leslie Gower type predator prey model with Michaelis-Menten type prey harvesting. There are also some paper [24,25] where harvesting is considered either prey or predator or both species. Lev et al. [24] studied the impact of harvesting on both Phytoplankton and Zooplankton population. Again, Jana et al. [25] investigated the effects of harvesting on predator population. These studies motivate us to consider the harvesting on the Fish species.

The paper is organized as mathematical model is formulated in Section 2. Boundedness of all solutions of the proposed system is analyzed in Section 3. The existence of different equilibrium points has been analyzed in Section 4. The stability analysis of our proposed system around the equilibrium point has been given in Section 5. Hopf bifurcation point is calculated in Section 6 with respect to some important parameters. In Section 7, Optimal control theory is applied to determine optimal harvesting rate. We have presented numerical simulation results in Section 8. Finally, in last section we have presented some of the main outcomes of our work.

2. Model formulation

For the study of the effects of Environmental toxin on the predator-prey dynamics, a three species predator-prey interaction model among Phytoplankton (P), Zooplankton (Z) and Fish (F) has been considered in the presence of Environmental toxin (E_T). It is assumed that Zooplankton consumes only Phytoplankton. The growth rate of Phytoplankton is considered as logistic form. Again, different Phytoplankton species produce some toxin such as Alexandrium sp., Amphidinium carterae, Chrysochromulina polylepis, Cooliamonotis, Dinophysis spKeeping etc and these toxins makes some death on Zooplankton population. It is also assumed that Fish consumes Phytoplankton as well as Zooplankton. The harvesting is considered on the Fish species only. It is assumed that the Environmental toxin (E_T) increases in marine water through different activities such as chemicals, particles, industrial, agricultural and residential waste, noise, or the spread of invasive organisms. Toxic metals can also be introduced into marine food webs. These can cause a change to tissue matter, biochemistry, behaviour, reproduction, and suppress growth in marine life. Also, many animal consume a high fish meal or fish hydrolysate content. In this way, marine toxins can be transferred to land animals and appear later in meat and dairy products. From the experimental study, it is proved that Environmental toxin (E_T) decreases the growth rate of Phytoplankton population. During predation these toxin are transferred to the Zooplankton and in this way it reaches in higher trophic level in a food chain. Keeping the above assumptions in mind, we extend the model of Panja and Mondal [4] in the presence of environmental toxin as follows: (Table 1).

Table 1
Biological/ecological meaning of parameters.

Symbol	Biological/ecological meaning
r	Intrinsic growth rate of Phytoplankton
γ	Contact rate between Phytoplankton and environmental toxin
γ_1	Proportionality constant
k	Environmental carrying capacity
β	Consumption rate of Phytoplankton by Zooplankton
α	Half saturation constant
γ_2	Consumption rate of Phytoplankton by Fish
β_1	Conservation coefficient from Phytoplankton to Zooplankton
d	Natural death rate of Zooplankton
ρ	Rate of toxin release by Phytoplankton
γ_3	Consumption rate of Zooplankton by Fish
S	Conservation coefficient from Phytoplankton to Fish
S_1	Conservation coefficient from Zooplankton to Fish
δ	Natural death rate of Fish
q	Catchability coefficient
E	Harvesting effort
A	Constant increase of Environmental toxin
d_1	Depletion rate of Environmental toxin

$$\left.\begin{aligned}\frac{dP}{dt}&=\frac{rP}{1+\gamma\gamma_1 PE_T}\left(1-\frac{P}{K}\right)-\frac{\beta PZ}{\alpha+P}-\frac{\gamma_2 PF}{\alpha+P}\\ \frac{dZ}{dt}&=\frac{\beta_1 PZ}{\alpha+P}-dZ-\frac{\rho PZ}{\alpha+P}-\frac{\gamma_3 ZF}{\alpha+P}\\ \frac{dF}{dt}&=\frac{SPF}{\alpha+P}+\frac{S_1 ZF}{\alpha+P}-\delta F-qEF\\ \frac{dE_T}{dt}&=A-\gamma PE_T-d_1E_T\end{aligned}\right\}\tag{1}$$

where $P(0)\geq 0$, $Z(0)\geq 0$, $F(0)\geq 0$, $E_T(0)\geq 0$.

3. Boundedness of solutions

In the following theorem, we show that the solutions of system (1) are bounded which would establish that the model formulation is ecologically meaningful.

Theorem 1. *The solutions of system (1) are all bounded.*

Proof. From the fourth equation of system (1), we have

$$\begin{aligned}\frac{dE_T}{dt}&=A-\gamma PE_T-d_1E_T\\ \frac{dE_T}{dt}&\leq A-d_1E_T\\ \frac{dE_T}{dt}&+d_1E_T\leq A\end{aligned}\tag{2}$$