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Hopf bifurcation control of hydro-turbine governing system with sloping ceiling tailrace tunnel using nonlinear state feedback

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A B S T R A C T

Aiming at improving the regulation quality of a hydro-turbine governing system with sloping ceiling tailrace tunnel, Hopf bifurcation control using nonlinear state feedback is studied. Firstly, the nonlinear mathematical model of a hydro-turbine governing system with sloping ceiling tailrace tunnel is presented. Then, a novel control strategy using nonlinear state feedback with polynomial functions is proposed, and Hopf bifurcation control of a hydro-turbine governing system using the proposed control strategy is described. Finally, the application and functional mechanism of the proposed control strategy are analyzed by comparison with the PID strategy. The results indicate that: The proposed nonlinear state feedback control strategy is able to make the frequency of a hydro-turbine unit return to the initial value (i.e. rated frequency), and the regulation quality and the response speed are obviously better than those under the PID strategy. The effect of the linear term of nonlinear state feedback control strategy is to modify the system's linear stability, in order to eliminate or delay an existing bifurcation. Altering the nonlinear term can change the stability of bifurcation solutions, for example, converting a Hopf bifurcation from subcritical to supercritical.

1. Introduction

The sloping ceiling tailrace tunnel is a kind of tailrace tunnel that is between the pressurized tailrace tunnel and the open channel [1,2]. It was proposed by Krivchenko in 1970 s and later successfully applied to the design of the Hoa Binh Hydropower Station in Vietnam. Nowadays, this kind of tailrace tunnel has been widely used all over the world, especially in China [3,4]. Many large hydropower stations, such as the Three Gorges Hydropower Station, the Ludila Hydropower Station and the Gongguoqiao Hydropower Station, adopted the sloping ceiling tailrace tunnel. Obvious technical and economic advantages have attracted general attention.

In the hydro-turbine governing process of a hydropower station with sloping ceiling tailrace tunnel, there is the reciprocating interface movement of the free surface-pressurized flow in the tailrace tunnel [5,6]. Then the operation characteristics of a hydro-turbine governing system with sloping ceiling tailrace tunnel are significantly different and much more complicated than those of a hydropower station with a pressurized tailrace tunnel [7,8]. The hydro-turbine governing system of a hydropower station with sloping ceiling tailrace tunnel is nonlinear [7]. And the regulation quality of this nonlinear dynamic system is a great challenge for the safety of the power grid and wide application of the sloping ceiling tailrace tunnel. But so far the control strategy used in the hydro-turbine governing system with sloping ceiling tailrace tunnel is usually the Proportion-Integration-Differentiation (PID) strategy [8,9], and high regulation quality is difficult to achieve when the PID strategy is used. Advanced control strategies have not been applied to this special hydro-turbine governing system.

In the past several decades, there has been rapidly growing interest in the bifurcation dynamics of nonlinear control systems, including controlling bifurcations and chaos [10–14]. The general goal of bifurcation control is to design a controller such that the bifurcation characteristics of a nonlinear system undergoing bifurcation can be modified to achieve certain desirable dynamical behavior [15]. Such bifurcation and chaos control techniques have been widely applied to solve physical and engineering problems. Various bifurcation control approaches have been proposed in the References [16–22]. In particular, for the problem of relocating an inherent Hopf bifurcation, a dynamic state-feedback control law incorporating a washout filter was proposed [17]. Since then, the washout filter-aided dynamic feedback control has been widely applied in controlling Hopf bifurcations in various nonlinear systems [23,24]. Later, a static state feedback control with polynomial functions was proposed [15]. A new robust adaptive hybrid controller for three-phase pulse-width modulated rectifiers to improve control performance was proposed by combining the second-order

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Nomenclature

Q	hydro-turbine unit discharge, m^3/s
α	ceiling slope angle of sloping ceiling tailrace tunnel, rad
h_f	head loss in penstock, m
H	hydro-turbine net head, m
N	hydro-turbine unit frequency, Hz
r	frequency disturbance
M_t, M_g	kinetic moment, resisting moment, N•m
e_h, e_x, e_y	moment transfer coefficients of turbine
e_g	load self-regulation coefficient
c	wave velocity of free surface flow, m/s
V_x	flow velocity of the interface of the free surface-pressurized flow, m/s
λ	cross-section coefficient of tailrace tunnel
B	width of sloping ceiling tailrace tunnel, m
Y	guide vane opening, mm
u	regulator output of governor
T_a	hydro-turbine unit inertia time constant, s
e_{qh}, e_{qx}, e_{qy}	discharge transfer coefficients of turbine
T_y	following mechanism inertia time constant, s

sliding mode with the adaptive gain super-twisting control law in the DC bus voltage control loop and state feedback adaptive control [25]. The control problem for static boost type converters using a high gain state feedback robust controller incorporating an integral action was investigated in [26]. Ref [27] considered the adaptive partial-state feedback stabilization for a class of stochastic high-order nonlinear systems with stochastic inverse dynamics and nonlinear parameterization, and the designed controller could guarantee that the equilibrium at the origin of the closed-loop system was globally stable in probability and all states can be regulated to the origin almost surely. In recent years, Hopf bifurcation theory has been introduced to researches on hydropower regulation systems [28–30], and the method is mainly used to determine the influence of the nonlinearity of governor and excitation system on the dynamics.

For a hydro-turbine governing system with sloping ceiling tailrace tunnel, this paper aims at improving the regulation quality by designing a controller using nonlinear state feedback with polynomial functions based on Hopf bifurcation theory. To achieve this goal, this paper is organized as follows. Firstly, the nonlinear mathematical model of a hydro-turbine governing system with sloping ceiling tailrace tunnel is presented in Section 2. Then, a novel control strategy using nonlinear state feedback with polynomial functions is proposed, and the details of the Hopf bifurcation control of a hydro-turbine governing system using the proposed control strategy are described in Section 3. The proposed control strategy is specifically designed for the hydro-turbine governing system with sloping ceiling tailrace tunnel and can satisfy the requirements of the load frequency control of the hydro-turbine governing system. Finally, the application and functional mechanism of the proposed control strategy are analyzed by numerical simulation comparison with the PID method in Section 4. In Section 5, the whole paper is summarized and conclusions are given.

2. Nonlinear mathematical model of the hydro-turbine governing system

The pipelines system of the hydropower station with sloping ceiling tailrace tunnel is shown in Fig. 1, and the structure diagram of the hydro-turbine governing system is shown in Fig. 2.

The dynamic equation of the penstock with sloping ceiling tailrace tunnel [7] is

$$h = -\frac{\lambda Q_0 V_x}{gH_0 cB\tan\alpha} q\frac{dq}{dt} - T_{ws}\frac{dq}{dt} - \left(\frac{2h_f}{H_0} + \frac{\lambda Q_0}{H_0 cB}\right)q \tag{1}$$

and the moment and discharge equations of the hydro-turbine [31] are, respectively

$$m_t = e_h h + e_x x + e_y y \quad \text{and} \tag{2}$$

$$q = e_{qh} h + e_{qx} x + e_{qy} y \tag{3}$$

The equation of the generator [32] is

$$T_a\frac{dx}{dt} = m_t - (m_g + e_g x) \tag{4}$$

and the equation of the following mechanism [33] is

$$\frac{dy}{dt} = \frac{1}{T_y}(u - y) \tag{5}$$

Note that: $h=(H-H_0)/H_0$, $q=(Q-Q_0)/Q_0$, $x=(N-N_0)/N_0$, $y=(Y-Y_0)/Y_0$, $m_t=(M_t-M_{t0})/M_{t0}$ and $m_g=(M_g-M_{g0})/M_{g0}$ are the relative deviations of corresponding variables. The subscript '0′ refers to the initial value. m_g is regarded as the load disturbance. Frequency disturbance is not considered. Therefore, we have $r=0$. The expressions for other variables are the same with those in [7].

Eqs. (1)–(5) can be integrated and yield the following three-dimensional nonlinear dynamic system:

$$\begin{cases} \dot{q} = \dfrac{-\left(\dfrac{2h_f}{H_0} + \dfrac{\lambda Q_0}{H_0 cB} + \dfrac{1}{e_{qh}}\right)q + \dfrac{e_{qx}}{e_{qh}}x + \dfrac{e_{qy}}{e_{qh}}y}{\dfrac{\lambda Q_0 V_x}{gH_0 cB\tan\alpha}q + T_{ws}} \\ \dot{x} = \dfrac{1}{T_a}\left[\dfrac{e_h}{e_{qh}}q + \left(e_x - \dfrac{e_h}{e_{qh}}e_{qx} - e_g\right)x + \left(e_y - \dfrac{e_h}{e_{qh}}e_{qy}\right)y - m_g\right] \\ \dot{y} = \dfrac{1}{T_y}(u - y) \end{cases} \tag{6}$$

Eq. (6) is the state equation of the hydro-turbine governing system.

3. Hopf bifurcation control of hydro-turbine governing system using nonlinear state feedback

3.1. Proposed control strategy using nonlinear state feedback with polynomial functions

For a n-dimensional nonlinear dynamic system, such as the system expressed by Eq. (6), its mathematical model can be transformed into the following form:

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu) \\ &\mathbf{f}: R^{n+1} \to R^n,\ \mathbf{x} \in R^n,\ \mu \in R \end{aligned} \tag{7}$$

where $\mathbf{x}$ is the state vector, μ is the scalar bifurcation parameter, and the vector field $\mathbf{f}(\mathbf{x}, \mu)$ is smooth in $\mathbf{x}$ and μ. Suppose that the system (7) has an equilibrium point at $\mathbf{x}=\mathbf{x}_E$, i.e. $\mathbf{f}(\mathbf{x}_E, \mu)=0$.

The goal of traditional Hopf bifurcation control [15] is to design a controller, given by:

$$\mathbf{u} = \mathbf{u}(\mathbf{x}, \mu) \tag{8}$$

so that $\mathbf{x}_E$ is unchanged, but the Hopf bifurcation point $(\mathbf{x}_E, \mu_c)$ is moved to a new position $(\tilde{\mathbf{x}}, \tilde{\mu})$, i.e. $(\tilde{\mathbf{x}}, \tilde{\mu}) \neq (\mathbf{x}_E, \mu_c)$. Hence, a necessary condition for the designed controller is obtained as

$$\mathbf{u}(\mathbf{x}_E, \mu) = 0 \tag{9}$$