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Homoclinic bifurcation and the Belyakov degeneracy in a variant of the Romer model of endogenous growth

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ABSTRACT

This paper explores the possibility of complex dynamics in a variant of the [29] model of endogenous growth. In particular, we derive the exact parametric configuration that allows for the emergence of a double-pulse homoclinic orbit, and the rise of globally indeterminate solutions, in the same area where local determinacy was found. Our results confirm that irregular patterns and oscillating solutions can be obtained along a subsidiary homoclinic orbit at which the periodic loop starts to double, so that the system might perpetually oscillate around the long run equilibrium, being thus confined in a stationary trapping region outside the neighborhood of the steady state. The economic implications of these results are finally discussed.

1. Introduction

As commonly pursued in the literature, the problem of determinacy of the equilibrium is mostly investigated from a local perspective, basically studying the saddle-path dynamics of a model only in the vicinity of the steady state solution. Unfortunately, local analysis is not able to provide a complete picture of the full dynamics exhibited by a nonlinear model. That is, the presence of a saddle-path stable (determinate) steady state does not exclude the existence of a more complicate dynamic scenario in the large, with multiple equilibrium trajectories possibly wandering towards an additional non-saddle equilibrium, or to a limit set with either regular or chaotic patterns (e.g., [5–9,12,20,24–28]). Hence, the concern on global indeterminacy cannot be neglected if we want to provide correct policy prescriptions that allow to locate the economy onto the optimal long run equilibrium path. A still scant but fast-growing literature is concentrating the attention on this issue (e.g., [2,3,10,12,13,23]).

This paper explores the possibility of complex dynamics in the [29] seminal contribution to the field of endogenous growth theory, in the variant proposed by Benhabib and Farmer [5] and Slobodyan [31] that extend the original model by introducing some degree of complementarity between intermediate capital goods. In particular, we derive the parametric configuration that allows for the emergence of a double-pulse homoclinic orbit, and the rise of globally indeterminate solutions, in the same area where local determinacy was found. Our results confirm that irregular patterns and oscillating solutions can be obtained along a subsidiary homoclinic orbit, in presence of a so-called Belyakov degeneracy, at which the periodic loop starts to double, so that the system might perpetually oscillate (with different periods of the generated cycles) around the long run equilibrium, perhaps without ever attaining the steady state again, being thus finally confined in an outer stationary trapping region (e.g., [4,16,18]).

The paper develops as follows. In Section 2, we present the model and study the long run properties of the three-dimensional vector field. In Section 3, we apply the standard mathematical procedure to construct the homoclinic orbit, and infer the region of the parameter space at which the Belyakov degeneracy occurs. An example is also provided to show the appearance of the double-pulse subsidiary orbit, which drives the economy to a global indeterminate scenario in the parametric area where local determinacy was found. A brief conclusive section reassesses the main findings of the paper. The Appendix A provides all calculations and necessary proofs.

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2. The modified Romer model

In this section, we describe a modified version of the [29] endogenous growth model, in line with the variant proposed by Benhabib and Farmer [5] and Slobodyan [31] that introduce some degree of complementarity between intermediate capital goods.

The economy is characterized by a continuum of firms operating in two different sectors, one devoted to production of capital goods and the other to research. Assume also that firms operating in the final goods sector are perfect competitors, whereas the market for intermediates is monopolistic.

Production of final good, Y, is realized through the following technology

$$Y = h^{\alpha} L^{\beta}\left(\int_0^A x(i)^{\frac{\gamma}{\xi}}\,di\right)^{\xi} \tag{1}$$

where h is the amount of skilled workers employed in the production sector, and L represents the amount of unskilled labor; $x(i)$ is the unit of intermediate goods indexed by an increasing level of knowledge, A. Moreover, $(\alpha, \beta)\in(0, 1)$ are the shares of human capital and unskilled labor in the final goods sector, respectively. Additionally, $\gamma = 1-\alpha-\beta$ is set to guarantee constant returns to scale in technology. Finally, $\xi \geq 1$ permits to introduce a degree of complementarity among intermediate capital goods, which extends the original setting described in [29], where $\xi = 0$.

Assuming that in equilibrium all intermediate goods are equally supplied, i.e. $x(i) = \bar{x}$, the amount of total capital in the economy is given by $K = \left[\eta \int_0^A x(i)di\right]_{x(i)=\bar{x}} = \eta A\bar{x}$, where η measures the units of intermediates, and therefore

$$Y = \eta^{-\gamma} h^{\alpha} L^{\beta} A^{\xi-\gamma} K^{\gamma} \tag{2}$$

is the total output. Hence, physical capital evolves over time according to

$$\dot{K} = Y - C = \eta^{-\gamma} h^{\alpha} L^{\beta} A^{\xi-\gamma} K^{\gamma} - C \tag{3}$$

being C the aggregate level of consumption.

Following [29], and setting the price of intermediates at their marginal revenue, we easily obtain the inverse demand function

$$p(i) = \frac{\partial Y}{\partial x(i)} = \gamma h^{\alpha} L^{\beta}\left(\int_0^A x(i)^{\frac{\gamma}{\xi}}\,di\right)^{\xi-1} x(i)^{\frac{\gamma}{\xi}-1} \tag{4}$$

and the profit of firm i

$$\pi(i) = p(i)x(i) - r\eta x(i) \tag{5}$$

where $p(i)x(i)$ measures the total revenue from selling the intermediate goods at price $p(i)$, and $r\eta x(i)$ is the total cost of renting them, at the rate r.

Optimization in final good firms requires that price is also equal to the marginal cost, $p(i) = r\eta$. Hence, using (4), we derive

$$r = \frac{\gamma^2 \eta^{-\gamma}}{\xi} K^{\gamma-1} A^{\xi-\gamma} h^{\alpha} L^{\beta} \tag{6}$$

and obtain

$$\pi = \frac{\eta(\xi-\gamma)}{\gamma} r\bar{x} \tag{7}$$

which is in fact the aggregate profit.

In the research sector, accumulation of knowledge is driven by

$$\dot{A} = \delta(1-h)A \tag{8}$$

where, $(1-h)$ is the percentage of skilled workers enrolled in the sector, and $\delta > 0$ is a scale parameter.[1]

[1] To simplify the notation used in [5] and [31], we set here the total amount of skilled labor to unity, $H = 1$.

Assuming free entry, the price of new (technologically augmented) goods produced in the research sector, P_A, must equal the net present value of profits

$$P_A = \int_0^{\infty} \pi(\tau) e^{-\int_t^{\tau} r(s)ds} d\tau \tag{9}$$

Using (6) an (7) in the time-derivative of (9), the law of motion of P_A reads

$$\dot{P}_A = rP_A - \pi = \left[r - \frac{\delta\gamma(\xi-\gamma)}{\alpha\xi} h\right] P_A \tag{10}$$

Notice also that, given the numeraire price of final output, no arbitrage condition between wages of skilled workers employed in both sectors implies that $P_A \frac{\partial \dot{A}}{\partial h} = \frac{\partial Y}{\partial h}$. Then, by using (2) and (8),

$$P_A = \frac{\alpha\eta^{-\gamma}}{\delta} h^{\alpha-1} L^{\beta} A^{\xi-\gamma-1} K^{\gamma} \tag{11}$$

Let preferences be described by the standard CES utility function, $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$. Hence, the representative agent has to solve the following optimization problem

$$\begin{aligned} &\max \int_0^{\infty} \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \\ &\text{s.t.} \\ &\dot{K} = \eta^{-\gamma} K^{\gamma} A^{\xi-\gamma} h^{\alpha} L^{\beta} - C \\ &\dot{A} = \delta(1-h)A \\ &\dot{P}_A = \left[r - \frac{\delta\gamma(\xi-\gamma)}{\alpha\xi} h\right] P_A \end{aligned} \tag{$\mathcal{P}$}$$

where $\sigma \neq 1$ is the inverse of the intertemporal elasticity of substitution, and ρ in the social discount rate. The full set of parameters is then $\theta \equiv (\alpha, \beta, \gamma, \delta, \eta, \rho, \xi, \sigma) \in \mathbb{R}^7_{++} \times \mathbb{R}_+ - \{1\}$.

Application of the standard Keynes–Ramsey rule requires that

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma} \tag{12}$$

that is also by (6)

$$\frac{\dot{C}}{C} = \frac{\gamma^2\eta^{-\gamma}}{\sigma\xi} h^{\alpha} L^{\beta} A^{\xi-\gamma} K^{\gamma-1} - \frac{\rho}{\sigma} \tag{13}$$

The optimal control problem $\mathcal{P}$ is characterized by four differential equations (3), (8), (10) and (13) in the four unknowns C, K, A, and P_A. It can be easily reduced by taking log-derivatives of (6) and (11), and introducing the new non-predetermined variable $q = \frac{C}{K}$, which denotes a stationarizing transformation of the consumption to capital ratio.

A bit of mathematical manipulation allows to derive the following three-dimensional system of first order differential equations, in the simplified version derived by [31]. That is,

$$\begin{aligned} \dot{r} &= \frac{r}{(1-\alpha)}\left[(\xi-1+\beta)\delta(1-h) - \beta\left(\frac{\xi}{\gamma^2}r - q\right) - \alpha\left(r - \frac{\delta}{\Lambda}h\right)\right] \\ \dot{h} &= \frac{h}{(1-\alpha)}\left[(\xi-1-\gamma)\delta(1-h) + \gamma\left(\frac{\xi}{\gamma^2}r - q\right) - \left(r - \frac{\delta}{\Lambda}h\right)\right] \\ \dot{q} &= q\left[\frac{r-\rho}{\sigma} - \frac{\xi}{\gamma^2}r + q\right] \end{aligned} \tag{S}$$

where $\Lambda = \frac{\alpha\xi}{\gamma(\xi-\gamma)}$ to compact notation.

Let

$$r^* = \frac{\sigma}{\sigma-1}\left[\frac{\delta}{\Lambda}h^* - \delta(1-h^*) - \frac{\rho}{\sigma}\right] \tag{14a}$$

$$h^* = \frac{\Lambda}{\delta} \frac{\delta[\sigma(\xi-\gamma)-(\xi-1)] + \rho(1-\gamma)}{\Lambda[\sigma(\xi-\gamma)-(\xi-1)] + (1-\gamma)} \tag{14b}$$