Chaos, Solitons and Fractals 104 (2017) 461–467

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Electronic implementation of a practical matched filter for a chaos-based communication system

Frank T. Werner[a,b,*], Benjamin K. Rhea[b], R. Chase Harrison[b], Robert N. Dean[b]

[a] School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA
[b] Department of Electrical and Computer Engineering, Auburn University, Auburn, AL 36849, USA

ARTICLE INFO

Article history:
Received 26 May 2017
Revised 14 August 2017
Accepted 29 August 2017

Keywords:
Chaos electronics
Matched filter
Communication systems

ABSTRACT

Presented is an electronic implementation of a matched filter intended for chaos-based communication systems. While implementing the transmitter side of such systems is relatively trivial, the receiver has proven challenging to develop. Most chaotic systems lack a known fixed basis function, making it difficult to develop a matched filter for them. Instead, their receivers rely on more complicated or less effective techniques to compensate for the presence of noise. However, a previously developed manifold piecewise linear chaotic system has been shown to have an exact analytic solution. This solution has enabled the development of a matched filter for use in any communication system based on this chaotic system. The original communication system operated at a fundamental frequency of 84 Hz, much too low for any practical applications. Therefore, newer communication systems have been designed to operate at higher frequencies. In this work, the performance of this matched filter has been evaluated with a 18.4 kHz chaotic oscillator.

1. Introduction

Currently, chaos electronics have been used in a number of applications, including random number generation [1], noise generation [2], radar [3], device characterization [4], robotics [5], and communications [6–10]. Chaos-based communication systems have received particular interest because of the advantages chaotic systems can provide for improving security and performance. These advantages are the result of the unique properties inherent to chaotic systems.

In general, chaotic systems are deterministic; however, their aperiodic long-term behavior and sensitivity to initial conditions makes it difficult to predict their behavior for more than a short period of time [11]. Furthermore, chaotic waveforms possess a continuous power spectral density from DC up to the fundamental frequency of oscillation. Due to these properties, chaotic waveforms give the appearance of noise when observed over a prolonged period of time. As a result, it can be difficult for anyone other than the intended recipient to detect a signal from a chaos-based communication system [12–14].

Futhermore, while the behavior of chaotic systems is complex, many such systems are relatively simple to implement in electronics [6]. As a result, chaos-based communication systems can be much simpler than communication systems relying on other techniques for improving security. Another advantage is that the wide bandwidth of chaotic waveforms improves the signal's robustness against disturbances impacting a narrow frequency range, such as filtering from multipath propagation and inference from periodic signals [10].

Implementing a chaos-based commincation system can be challenging. While it is relatively simple to develop a transmitter, it is difficult to develop a receiver that can detect the chaotic waveform in the presence of additive white Gaussian noise (AWGN) [15]. In a conventional communication system, a matched filter is the optimal method for detecting a signal in the presence of AWGN. However, the basis function of the waveform needs to be known in order to implement the matched filter. Most chaotic systems do not have a known analytic solution, meaning that a matched filter cannot be readily developed for them [16].

Unlike most chaotic systems, an analytic solution has been derived for the manifold piecewise linear chaotic system originally described by Saito and Fujita [17]. Later work showed that this system can be characterized as a linear convolution of a sequence of binary symbols and a fixed basis function [16]. From these findings, Corron et al. developed a chaotic oscillator and matched filter based on this chaotic system [18,19]. However, their oscillator operated at a fundamental frequency of 84 Hz, severely limiting its range of applications.

* Corresponding author.
E-mail address: ftw0001@auburn.edu (F.T. Werner).

http://dx.doi.org/10.1016/j.chaos.2017.08.029

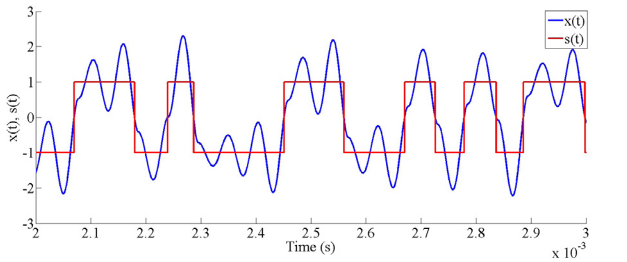

Fig. 1. An example of the chaotic system's output, $x(t)$ (in blue) and the nonlinear forcing function, $s(t)$, (in red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

To enlarge the range of potential applications, the fundamental frequency of the chaotic oscillator needed to be increased by several orders of magnitude. This goal led to the development of the chaotic oscillators designed by Beal and co-workers [20,21]. These oscillator designs have fundamental frequencies ranging from 18.4 kHz to 1.7 MHz, making them much more practical. However, before the new chaotic oscillators could be used in a communication system, a new matched filter needed to be developed to operate at the same fundamental frequency as the new oscillators. Presented is a hardware implementation of this new matched filter. Its performance with a 18.4 kHz chaotic oscillator has been evaluated by both simulation and hardware testing.

2. Background: The chaotic system

2.1. Saito's manifold piecewise linear chaotic system

As stated above, both the original and new chaotic oscillators are based on the manifold piecewise linear chaotic system originally defined by Saito and Fujita. The advantage of this system is that an exact analytic solution has been developed. This system is a synthesis of a linear second-order continuous time differential equation with a nonlinear discrete forcing function [17]. The continuous components of the system are represented by the second-order linear differential equation defined in (1).

$$\ddot{x}(t) - 2\beta\dot{x}(t) + (\omega^2 + \beta^2)x(t) = (\omega^2 + \beta^2)s(t). \tag{1}$$

In the equation, $x(t)$ is the system's ouput, ω is the fundamental radial frequency, and β is equivalent to a positive Lyapunov exponent [16]. In order for the system to remain in chaotic motion, $0 < \beta \leq \ln(2)$. In (1), $s(t)$ is the nonlinear forcing function represented by the piecewise function in (2).

$$s(t) = \begin{cases} +1 & x(t) \geq 0 \\ -1 & x(t) < 0 \end{cases} \tag{2}$$

The continuous portion of the chaotic system has an unstable response due to the positive Lyapanov exponent. The oscillation is centered on the instantaneous equilibrium point defined by $s(t)$. The magnitude of the oscillation increases until the output satisfies the guard condition. The guard condition occurs when there is an instantaneous zero crossing of the oscillation and the derivative of the oscillation. Afterwards, the value of $s(t)$ switches, as defined in (2), and the oscillation continues around the new value of $s(t)$. Fig. 1 shows an example of the system's output, $x(t)$, and $s(t)$. The output of the chaotic oscillator can be controlled using arbitrarily small perturbations to stablize periodic orbits [22,23]. This process can be used to encode information into the chaotic waveform before it is transmitted. The encoded information, or symbolic content, is represented by the forcing function, $s(t)$ [6,24]. Since $s(t)$ is part of the oscillator's feedback path, controlling $s(t)$ influences the following $x(t)$, shown in (3).

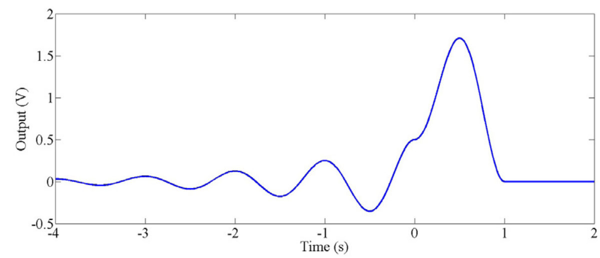

Fig. 2. The basis function for the chaotic system when $\beta = \ln(2)$.

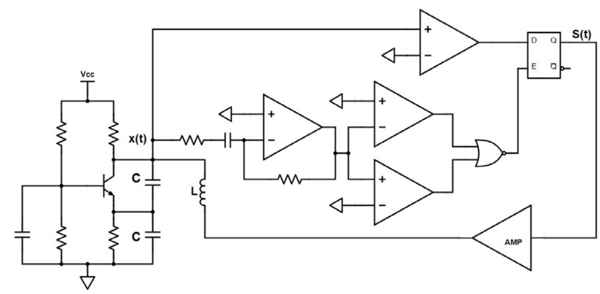

Fig. 3. Generalized schematic of the chaotic oscillator.

2.2. Basis function

The analytic solution of the chaotic system can be written as a linear convolution of the symbolic content and a fixed basis function by solving (1) for $x(t)$, as shown in (3).

$$x(t) = \sum_{m=-\infty}^{\infty} s_m P(t-m) \tag{3}$$

In (3), $P(t)$ is the basis function and s_m represents the symbolic content of the system. The symbolic content modulates the basis function centered at time, $t = m$ [19]. The equation for the basis function, $P(t)$, is represented in (4). As an example, the plot of the basis function's waveform when $\beta = \ln(2)$ is shown in Fig. 2.

$$P(t) = \begin{cases} (1 - e^{-\beta})e^{\beta t}(cos(\omega t) + sin(\omega t)) & [t] < 0 \\ 1 - e^{-\beta(t-1)}(cos(\omega t) + sin(\omega t)) & 0 \leq [t] < 1 \\ 0 & [t] \geq 1 \end{cases} \tag{4}$$

2.3. Overview of the chaotic oscillator circuit

The schematic of the chaotic oscillator is shown in Fig. 3. This circuit was developed by B. K. Rhea et. al. based on the manifold piecewise linear system described above [25]. It is comprised of a LC resonant circuit, a common base amplifier, and a feedback network. As shown in Fig. 3, $x(t)$ is generated by the resonant circuit, while $s(t)$ is generated by the feedback network. When in operation, the common base amplifier functions effectively as a negative resistance to the circuit. The output of the resonant circuit is sampled by the feedback network to generate s(t), which is in turn fed back into the resonant circuit. As labeled in the figure, the output of the oscillator, $x(t)$, is generated by the resonant circuit. Afterwards, $s(t)$ is fed into the resonant circuit in order to generate $x(t)$. The fundamental frequency of the oscillator is set by the values of C and L in Fig. 3 and can be calculated using (5). For the 18.4 kHz oscillator, $C = 1$ µF and $L = 150$ µH.

$$f = \frac{1}{2\pi\sqrt{2C * L}} \tag{5}$$