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Two-stage method for fractal dimension calculation of the mechanical equipment rough surface profile based on fractal theory

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A B S T R A C T

The determination of fractal dimension of rough surface profile curve is important for characterizing the fractal features of rough surface microscopic topography. There are many methods to calculate the fractal dimension, such as the power spectrum method (PSM), the structure function method (SFM), the variation method, the R/S analysis method, the wavelet transform method and etc., among which the PSM and SFM are widely used methods. This study aims to improve the computational accuracy of the fractal dimension of the profile curve. For this purpose, the two-stage method based on PSM and SFM are proposed. Firstly, we analyze the principle of calculating the fractal dimension of profile curve using PSM and SFM. Then, based on PSM and SFM, we propose a two-stage method for determining the fractal dimension of profile curve. Simulation results show that the two-stage method for fractal dimension of profile curve can greatly reduce the error compared with the original PSM and SFM. Finally, the fractal dimensions of the profile curve of the cuboid specimen are calculated by the original PSM and SFM and the two-stage method respectively. The experimental results show that the proposed method provides more precise results for determining the fractal dimension.

1. Introduction

In mechanical equipment, there are many phenomena in the rough surface such as wear, fatigue and so on, the description of the microscopic characteristics of rough surface is of great significance to the study of the contact problem of mechanical equipment. The traditional methods of surface microscopic feature's description based on statistical parameters have scale dependence, that is, for the same contact surface, different statistical parameters are obtained if the resolution of the measuring instrument is different or the sampling length is different. So the result of the contact model is not unique to the rough surface. Fractal theory has been introduced in the contact mechanics to perform scale-independent analysis, which provides a feasible way to solve the problem above. The main idea of this theory is that in some phenomena and processes, some aspects of the local are similar to that of the whole. This theory was proposed by Mandelbort and applied to the study of the British coastline [1]. A fractal is a mathematical method, the parameters do not change with the resolution of the instrument or the length of the sample. In addition, the result of the contact model is unique to the rough surface.

Currently, there are a lot of researches on the contact model based on fractal theory. Majumdar and Bushan established the elastic–plastic contact model of fractal rough surfaces (MB fractal contact model) [2], the fractal theory is applied to the contact analysis of two rough surfaces for the first time. Wang and Komvopoulos established a new fractal contact model (WK fractal contact model), which can study the elastic/elastoplastic contact and heat transfer analysis [3]. Yan et al. for the first time used fractal theory to study the contact problem from the view of three dimensions [4]. According to Yahav et al., the deformation of the asperity is transformed from elastic deformation to plastic deformation. A modified elastic–plastic contact model of a single fractal asperity (ME model) is proposed based on the MB model [5]. The scholars have done some works on viscoelastic wear analysis based on MB model and its modified models [6–8]. Qi et al. studied on the calculation of contact stiffness of two cylinders based on fractal model [9]. Renato et al. studied on the contact condition of two metal surfaces with fractal theory [10]. Iasef et al. explored the application of fractal geometry in architecture and art [11]. Yuan et al. used the fractal theory to characterize engineering surfaces and wear particles [12]. Carpinteri et al. studied the fa-

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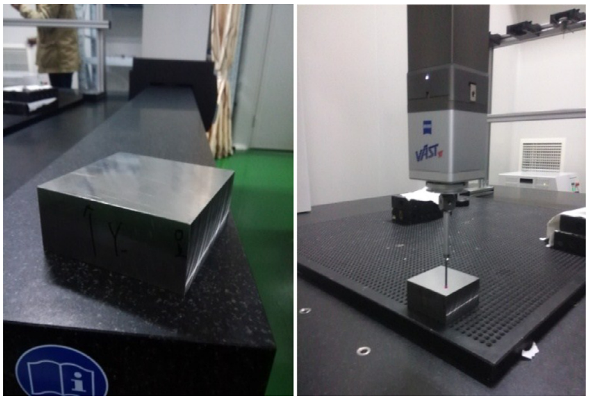

Fig. 1. The cuboid specimen and ZEISS three coordinate measuring instrument.

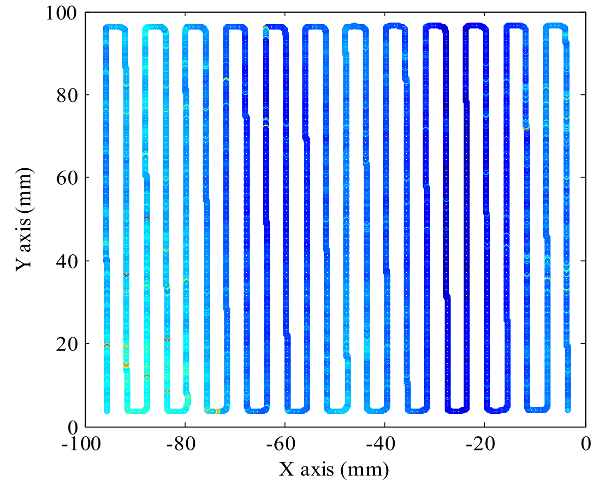

Fig. 2. The mining path of the ZEISS three coordinate measuring instrument.

tigue growth based on fractal theory [13]. You et al. studied a static friction model for the contact of fractal surfaces contact of fractal surfaces [14]. Zou et al. developed a random model based on fractal geometry theory to calculate the TCC of two rough surfaces in contact [15]. Chen et al. proposed a matching filtering method based on fractal model for separating magnetic anomalies [16]. Richen et al. studied the hydraulic characteristics of rock fracture network based on fractal theory [17].

In summary, fractal theory has been widely used in engineering and physics, such as construction field, rock fracture network analysis, surface contact problems, and viscoelastic wear analysis etc. However, the calculation of fractal dimension of the surface profile is of great importance in fractal theory. Several studies of the fractal dimension have been presented, but limited. Liu et al. characterized the spheroidization grade and the strength of 15CrMo steel through determining fractal dimension [18]. Ray et al. studied the correlation between fractal dimension and impact energy in ahigh strength low alloy steel [19].

At present, the commonly used methods for calculating fractal dimension, such as power spectrum method and structure function method, have large errors. Based on the two original methods, this paper aims to propose the two-stage method, which will greatly improve the computational accuracy of the fractal dimension of the profile curve.

The rest of the paper is organized as follows: after an introduction, we present the experimental materials and analyze the principle of calculating the fractal dimension of profile curve using PSM and SFM. Then, based on PSM and SFM, we each propose a two-stage method for determining the fractal dimension of profile curve respectively in Section 2. A detailed comparison and further experimental verification are presented in Section 3. A conclusion is drawn in Section 4.

2. Material and methods

2.1. Material

The studied material is 45 steel with 50HRC hardness. It can be seen from the Ref. [20] that the surface topography of cuboid specimen can be described by fractal theory. The experiments were conducted in three coordinate measuring instrument by selecting the cuboid specimen with 100 mm side length and 40 mm height, as shown in Fig. 1. The surface topography of cuboid specimen is measured by ZEISS three coordinate measuring instrument, which is to measure the height value of multiple points on the whole surface. The surface sampling path is shown in Fig. 2, each vertical curve collects 2300 points. The sampling interval is 0.043 mm.

2.2. Methods

2.2.1. The traditional methods for fractal dimension of profile curve and its error analysis

The commonly used method of fractal dimension is power spectrum method, its principle is to use Weierstrass–Mandelbrot function (WM function) to simulate the fractal profile curve, then the power spectrum is taken as the measure and the frequency is taken as the scale, finally, the fractal dimension is calculated by the slope of the straight line in the logarithmic coordinates.

Weierstrass (1872) derived the famous everywhere continuous but everywhere non differentiable function, that is, WM function, the fractal profile curve can be characterized by using the real part of WM function as follows:

$$z(x) = G^{D-1}\sum_{n=n_l}^{\infty}\gamma^{-(2-D)n}\cos(2\pi\gamma^{n}x), \quad 1 < D < 2, \gamma > 1 \tag{1}$$

where D is the theoretical fractal dimension, G is the height scale parameter independent of frequency, the term is called fractal roughness, x is horizontal coordinates, γ^n is spatial frequency for random profiles, n_l is the ordinal number corresponding to the lowest cut-off frequency.

The power spectrum function of two-dimensional WM function can be obtained by Ref. [2]

$$P(w) = \frac{1}{w^{5-2C}\ln\gamma} \tag{2}$$

where $P(w)$ is the power spectrum, C is the fractal dimension calculated by power spectrum method, w is the frequency.

The profile curve can be simulated when D is certain in Eq. (1), as shown in Fig. 3(a). Although the fractal dimension of the profile curve can be obtained by power spectrum method, as shown in Fig. 3(b), the calculated value is 1.6064, which is much different from the actual value of 1.4. A large number of simulation results also show that the fractal dimension calculated by the power spectrum method has a large error with the real dimension, as shown in Fig. 4. The reason that is, there are three methods calculating the 2D power spectrum density (2D PSD): (1) calculate the 2D PSD based on Fourier transform of autocorrelation function, (2) calculate the 2D PSD using filtering technology, (3) cal-