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Stochastic sensitivity analysis of nonautonomous nonlinear systems subjected to Poisson white noise

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ABSTRACT

The stochastic sensitivity function (SSF) method is extended to estimate the stationary probability distribution around periodic attractors of nonautonomous nonlinear dynamical systems subjected to Poisson white noise in this paper. After deriving the stochastic sensitivity functions of period-*N* cycle of mapping systems based on the characteristic of Poisson process, non-autonomous dynamical systems around periodic attractors are converted to mapping systems by constructing a stroboscopic map, and then the stochastic sensitivity functions of periodic attractors of nonautonomous nonlinear systems can be obtained by adopting the results of mapping systems. It is found that the stochastic sensitivity functions depend on the product of noise intensity and the arrival rate of Poisson processes. To illustrate the validity of the proposed method, a Henon map driven by Poisson processes and a Mathieu–Duffing oscillator under Poisson white noise are studied.

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1. Introduction

Random disturbance and nonlinearity widely exist in real systems [1–5]. The interplay between random disturbance and nonlinearity in dynamical systems can give rise to different kinds of phenomena like noise-induced enhancement [6], noise-induced synchronization [7], noise-induced transitions [8–10], noise-induced chaos [11,12], noise-induced intermittency [13,14], noise-enhanced stability [15], stochastic resonance [16,17] and anomalous diffusion [18].

A class of random excitations is modeled as Gaussian white noise (GWN). However, it is inappropriate to use Gaussian white noise to represent the random excitations which are characterized by impulsive loading, such as wind buffeting of airplane tail [19], wave action on a ship [20], moving loads traveling on a bridge [21], earthquakes shake on structures [22]. These kinds of random excitations are often modeled as Poisson white noise (PWN) [23].

The probability density function (PDF) of the response of nonlinear dynamical systems subjected to white noise is governed by the Fokker–Planck–Kolmogorov (FPK) equation [24]. However, in general, the FPK equation is very hard to solve directly. Therefore, many approximate and numerical approaches are proposed, such as equivalent linearization [25,26], stochastic averaging method [27,28], exponential-polynomial closure method [29,30], path integral method [31,32], finite element method [33,34], Monte Carlo simulation [35,36], etc.

For nonlinear dynamical systems subjected to GWN, the stochastic sensitivity function (SSF) method has been put forward based on the quasipotential theory and used for the probabilistic description of the stochastic attractor [37]. Since the SSF can be utilized to construct confidence domains which reflect the main features of spatial arrangement of random states under small noise, it has been applied to study the phenomena which is caused by the noise acting on nonlinear dynamical systems, such as stochastic bifurcations [38], noise-induced transitions [39] and noise-induced chaos [40].

Though the SSF technique has been widely used to analyze the response of nonlinear dynamical systems under GWN [41,42], to the best of our knowledge, it has not been employed to the response analysis of nonlinear systems under PWN. To describe the statistical characteristics of the stochastic attractor of nonautonomous nonlinear dynamical systems driven by PWN, the SSF method is first extended to the discrete nonlinear systems under PWN in this paper. Then, utilizing stroboscopic sections, periodic attractors of nonautonomous nonlinear dynamical systems are discretized into period-*N* cycles of the corresponding nonlinear mapping systems. On the basis of the results of mapping systems, the SSF of nonautonomous nonlinear dynamical systems can be derived. Furthermore, the dependence of the SSF on the noise intensity and the arrival rate of PWN is obtained.

This paper is organized as follows. In Section 2, the algorithms to derive the SSFs of periodic attractors of a nonautonomous non-

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linear dynamical system and a nonlinear mapping system are given. In Sections 3 and 4, the Henon mapping and the Mathieu–Duffing oscillator under PWN are studied to illustrate the method proposed in Section 2. Finally, conclusions are shown in Section 5.

2. Stochastic sensitivity function of periodic attractors of nonlinear systems driven by Poisson white noise

Consider a stochastic system

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, t) + \sigma(t)\xi(t) \tag{1}$$

where $\mathbf{X}$ is an n-dimensional state vector, $\mathbf{f}(\mathbf{X}, t)$ is a smooth n-dimensional vector-valued function on $\mathbf{X}$ and time t, $\sigma(t)=\{\sigma_1(t), \sigma_2(t), \ldots, \sigma_m(t)\}$ is an $n \times m$ matrix-valued function, and $\xi(t) = \{\xi_1(t), \xi_2(t), \ldots, \xi_m(t)\}^T$ is an m-dimensional uncorrelated Poisson white noise which is referred to as the formal derivative of the compound Poisson process

$$\mathbf{C}(t) = \{C_1(t), \ldots, C_m(t)\} = \left\{ \sum_{i=1}^{N_1(t)} Y_{1i}U_1(t - t_i), \ldots, \sum_{i=1}^{N_m(t)} Y_{mi}U_m(t - t_i) \right\}.$$

Here, for each $j \in \{1, 2, \ldots, m\}$, $N_j(t)$ is a Poisson counting process with rate λ_j, $U_j(t - t_i)$ is the unit step function at t_i, and Y_{ji} is a random variable which stands for the intensity of the ith impulse for the jth Poisson counting process $N_j(t)$. Assume that the sequence of random variables $\{Y_{ji}, i = 1, 2, \ldots\}$ is independent and identically distributed. Then,

$$E[d\mathbf{C}] = \{\lambda_1 E[Y_{11}]dt, \ldots, \lambda_m E[Y_{m1}]dt\},$$

$$Var[d\mathbf{C}] = \begin{pmatrix} \lambda_1 E[Y_{11}^2]dt & & 0 \\ & \ddots & \\ 0 & & \lambda_m E[Y_{m1}^2]dt \end{pmatrix},$$

If $Y_{ji} = 1$, then the compound Poisson process $C_j(t)$ is reduced to the Poisson counting process $N_j(t)$ and

$$E[d\mathbf{C}] = \{\lambda_1 dt, \ldots, \lambda_m dt\}$$

$$Var[d\mathbf{C}] = \begin{pmatrix} \lambda_1 dt & & 0 \\ & \ddots & \\ 0 & & \lambda_m dt \end{pmatrix},$$

Consider the corresponding deterministic system

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, t). \tag{2}$$

Assume that the system (2) has a periodic attractor with period T and $\boldsymbol{\varphi}_t(\mathbf{X})$ is the flow generated by vector field $\mathbf{f}(\mathbf{X}, t)$. Then for a state $\mathbf{X}_0$ on the periodic attractor corresponding to time t_0, we have $\varphi_T(\mathbf{X}_0) = \mathbf{X}_0$. Therefore, a stroboscopic map

$$\mathbf{X}_{k+1} = \varphi_{\Delta t}(\mathbf{X}_k) \tag{3}$$

where Δt is the sampling interval and k is a positive integer, is used to convert a continuous-time periodic trajectory $\mathbf{X}(t)$ into a discrete-time periodic trajectory $\{\mathbf{X}_k\}$. The stroboscopic sections are defined as

$$\Sigma_k = \{(\mathbf{X}, t) \in R^m \times R | t = t_0 + k\Delta t\}. \tag{4}$$

When $\Delta t = T/N$, where N is a positive integer, the periodic attractor of system (1) is discretized into a period-N cycle $\{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N\}$ by N stroboscopic sections $\{\Sigma_1, \Sigma_2, \ldots, \Sigma_N\}$. In this case, the map is called $1/N$-periodic stroboscopic map. The fourth order Runge–Kutta method is adopted in this paper to estimate the $1/N$ -periodic stroboscopic map (3) from t_k to t_{k+1}. That is to say, $\varphi_{\Delta t}(\mathbf{X}_k) = \mathbf{X}_k + (\mathbf{F}_{1,k} + 2\mathbf{F}_{2,k} + 2\mathbf{F}_{3,k} + \mathbf{F}_{4,k})\Delta t/6$ where $\mathbf{F}_{i,k}(i = 1, 2, 3, 4)$ stand for the integral coefficients of the kth iteration in the fourth order Runge–Kutta algorithm.

According to above results, the $1/N$ -periodic stroboscopic map of stochastic system (1) is approximated by the following expression

$$\bar{\mathbf{X}}_{k+1} = \varphi_{\Delta t}(\bar{\mathbf{X}}_k) + \sigma_k \Delta\mathbf{C}_k \tag{6}$$

where $\bar{\mathbf{X}}_k$ is the state of the stochastic system, $\boldsymbol{\sigma}_k$ stands for $\boldsymbol{\sigma}(t_k)$, and $\Delta\mathbf{C}_k = \mathbf{C}(t_{k+1}) - \mathbf{C}(t_k)$.

SSF of the periodic attractors of stochastic system (1) is derived below from the $1/N$ -periodic stroboscopic map (6) and the stochastic sensitivity analysis method of discrete systems.

Let $\mathbf{Z}_k = \bar{\mathbf{X}}_k - \mathbf{X}_k$ denote small deviations of states $\bar{\mathbf{X}}_k$ of stochastic system (6) from points $\mathbf{X}_k$ of the deterministic period-N cycle. Then the first approximation of stochastic system (6) at $\mathbf{X}_k$ on the stroboscopic section Σ_k is given by the following equation

$$\mathbf{Z}_{k+1} = \mathbf{A}_k\mathbf{Z}_k + \sigma_k \Delta\mathbf{C}_k \tag{7}$$

where $\mathbf{A}_k = \partial\varphi_{\Delta t}(\mathbf{X}, t_k)/\partial\mathbf{X}|_{\mathbf{X}=\mathbf{X}_k}$ and $\mathbf{A}_{k+N} = \mathbf{A}_k$. That is, $\mathbf{A}_k$ is an N-periodic matrix.

The first moment $\mathbf{m}_k = E[\mathbf{Z}_k]$ and the second central moment $\mathbf{V}_k = E[\mathbf{Z}_k\mathbf{Z}_k^T] - E[\mathbf{Z}_k]E[\mathbf{Z}_k^T]$ for the solution $\mathbf{Z}_k$ of the system (7) satisfy the following equations

$$\mathbf{m}_{k+1} = \mathbf{A}_k\mathbf{m}_k + \mathbf{D}_k \tag{8}$$

$$\mathbf{V}_k = \mathbf{A}_k\mathbf{V}_k\mathbf{A}_k^T + \mathbf{Q}_k \tag{9}$$

where $\mathbf{D}_k = \sigma_k E[\Delta\mathbf{C}_k]$ and $\mathbf{Q}_k = \sigma_k Var[\Delta\mathbf{C}_k]\sigma_k^T$. Assume that $\boldsymbol{\sigma}_k$ is an N-periodic matrix, then $\mathbf{D}_k$ and $\mathbf{Q}_k$ are also N-periodic matrices. Thus, the N consecutive iterations of systems (8) and (9) are

$$\begin{aligned} \mathbf{m}_{lN+2} &= \mathbf{A}_1\mathbf{m}_{lN+1} + \mathbf{D}_1 \\ \mathbf{m}_{lN+3} &= \mathbf{A}_2\mathbf{m}_{lN+2} + \mathbf{D}_2 \\ &\vdots \\ \mathbf{m}_{lN+N+1} &= \mathbf{A}_N\mathbf{m}_{lN+N} + \mathbf{D}_N \end{aligned} \tag{10}$$

$$\begin{aligned} \mathbf{V}_{lN+2} &= \mathbf{A}_1\mathbf{V}_{lN+1}\mathbf{A}_1^T + \mathbf{Q}_1 \\ \mathbf{V}_{lN+3} &= \mathbf{A}_2\mathbf{V}_{lN+2}\mathbf{A}_2^T + \mathbf{Q}_2 \\ &\vdots \\ \mathbf{V}_{lN+N+1} &= \mathbf{A}_N\mathbf{V}_{lN+N}\mathbf{A}_N^T + \mathbf{Q}_N \end{aligned} \tag{11}$$

where l is a positive integer. It is suggested that the expressions connecting $\mathbf{m}_{(l+1)N+1}$ with $\mathbf{m}_{lN+1}$ and $\mathbf{V}_{(l+1)N+1}$ with $\mathbf{V}_{lN+1}$ are

$$\mathbf{m}_{(l+1)N+1} = \mathbf{B}\mathbf{m}_{lN+1} + \mathbf{D} \tag{12}$$

$$\mathbf{V}_{(l+1)N+1} = \mathbf{B}\mathbf{V}_{lN+1}\mathbf{B}^T + \mathbf{Q} \tag{13}$$

where $\mathbf{B} = \mathbf{A}_N \cdots \mathbf{A}_2\mathbf{A}_1$, $\mathbf{D} = \mathbf{D}_N + \mathbf{A}_N\mathbf{D}_{N-1} + \ldots + \mathbf{A}_N \cdots \mathbf{A}_2\mathbf{D}_1$, and $\mathbf{Q} = \mathbf{Q}_N + \mathbf{A}_N\mathbf{Q}_{N-1}\mathbf{A}_N^T + \ldots + \mathbf{A}_N \cdots \mathbf{A}_2\mathbf{Q}_1\mathbf{A}_2^T \cdots \mathbf{A}_N^T$.

According to Ref. [43], if the inequality $\rho(\mathbf{B}) < 1$ holds, where $\rho(\mathbf{B})$ is the spectral radius of matrix $\mathbf{B}$, then the deterministic period-N cycle is exponentially stable. Therefore, the following theorem holds.

Theorem 1. Assume $\rho(\mathbf{B}) < 1$. Then

a) The N-periodic solution $\mathbf{g}_k$: $\mathbf{g}_{k+N} = \mathbf{g}_k$ of the system (8) is unique, where $\mathbf{g}_1$ is a unique solution of the following equation

$$\mathbf{g} = \mathbf{B}\mathbf{g} + \mathbf{D} \tag{14}$$

and $\mathbf{g}_2, \mathbf{g}_3, \ldots, \mathbf{g}_m$ are obtained by

$$\mathbf{g}_{k+1} = \mathbf{A}_k\mathbf{g}_k + \mathbf{D}_k, k = 1, 2, \ldots, N - 1. \tag{15}$$

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