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Modeling nonlinear wave regimes in a falling liquid film entrained by a gas flow

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The article studies nonlinear waves on a liquid film, flowing under the action of gravity in a known stress field at the interface. In the case of small Reynolds numbers, the problem is reduced to solving a nonlinear integro-differential equation for the film thickness deviation from the undisturbed level. The nature of branching of wave modes of the unperturbed flow with a flat interface has been investigated. The steady-state traveling solutions with wave numbers that are far enough from the neutral ones, have been numerically found. Using methods of stability theory, the analysis of branching of new families of steady-state traveling solutions has been performed. In particular, it is shown that, similarly to the case of the falling film, this model equation has solutions in the form of solitons-humps.

1. Introduction

The joint flow of liquid and gas is a classical problem of hydrodynamics. Flows of thin liquid films in the presence of countercurrent or cocurrent gas flows are often used in various technological applications. To date, various aspects of this problem have been considered in numerous works [1–13]. For example, such flows are used for cooling the miniature electronic equipment [10]. Nowadays the flowing liquid film has become a subject of a number of full review monographs, which have already become classical (see, e.g. [14–16]), but there are yet no such monographs devoted to the study of joint flows of viscous film and gas. Rather detailed reviews devoted to this problem may be found, e.g. [9,12,13]. The solution to this problem in a full conjugated formulation is associated with significant computational difficulties, therefore two stages of modeling are often distinguished: determining gas stresses on the film surface and subsequent calculation of wave evolution in the liquid. The possibility of the problem staging is justified in particular in [9].

It becomes possible, since the fluid velocity is usually much smaller than the characteristic gas velocity, so the boundary surface is believed to be rigid and immovable. In addition, because of the smallness of the film thickness, the impact of the boundary perturbation on gas velocities may be considered linear. Because of this, the problem of calculation of normal and shear stresses of gas on the surface is reduced to considering the impact of individual spatial harmonics. At the second phase of the joint flow study, the dynamics of nonlinear waves on the liquid film surface is investigated.

At such staging, the problem both for gas and liquid films was considered in various approximations: from laminar flow [2,5,6,11,13] to various models of the turbulent gas flow [2,6,8,9,12]. To describe the film flow there are also many models: from integral models of flows [2,6,11] to full Navier–Stokes equations [8]. At that, both Cartesian [2,8] and various curvilinear coordinates [9,12,17,18] were considered. Here we will mainly limit to referring to the works, directly used during the implementation of this study. Here, we are considering the second stage of the study, i.e. modeling the dynamics of nonlinear waves on liquid film, entrained by a cocurrent gas flow and falling under the action of gravity on the vertical plane, in a known stress field at the interface. In the same statement the problem is addressed in [12]. There, for the case of moderate Reynolds numbers, the steady-state traveling solutions were built, including soliton solutions for the system of two equations for film thickness and liquid flow rate.

2. Problem statement

In [18] for the system of hydrodynamic equations written in tensor form, invariant to coordinate systems, for the considered

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flow in the case of small flow rates (Re ∼ 1) an evolution equation for the film thickness h was obtained:

$$h_t + \frac{\mathrm{Re}}{Fr}h^2 h_x + \mathrm{Re}\tau_0 h h_x + \varepsilon\frac{\partial}{\partial x}\left(\frac{1}{3}\varepsilon \mathrm{Re} W h^3 h_{xxx} + \frac{2}{15}\frac{\mathrm{Re}^3}{Fr^2}h^5 h_x (h+\tau_0 Fr) + \frac{1}{2}\mathrm{Re} h^2 \tau_0 \int \hat{h}_k k\tau(k)\, e^{ikx} dk\right) = 0 \quad (1)$$

Here, $\mathrm{Re} = \rho h_0 u_0/\mu$ is the Reynolds number, $W = \sigma/\rho l_0 u_0^2$ is the Weber number, $Fr = u_0^2/g h_0$ is the Froude number, $\varepsilon = h_0/l_0$ is the ratio of specific film thickness h_0 to the characteristic wavelength l_0. In addition, in Eq. (1) and dimensionless complexes, the characteristic scales of velocity u_0 and time l_0/u_0 were used. Here, σ is the coefficient of surface tension, ρ is the density, μ is the dynamic viscosity of fluid, g is the acceleration of gravity, τ_0 is the unperturbed component of gas shear stresses on the film surface, $\tau(k) = \tau_r(k) + i\tau_{im}(k)$ are the Fourier components of gas shear stresses, conditioned by the interface curvilinearity, and $\hat{h}(k,t)$ are the Fourier components of the surface form expansion:

$$\hat{h}(k,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} h(x,t)\, e^{-ikx} dx,$$

Emphasize that in deriving Eq. (1) the approximation of small Reynolds numbers (Re ∼ 1) was used, and the Weber number was assumed large $W\varepsilon \sim 1$. Eq. (1) accurate to notations coincides with the equation obtained in [9]. In addition, some differences in the coefficients are due to the fact that it is the countercurrent flow of gas that was considered in [9], and there $\tau_0 < 0$.

In the case of spatially periodic solutions of Eq. (1), the integral term is replaced with the corresponding series.

Restricting to perturbations of small but finite amplitude, introducing slow and fast times into consideration, and using the transform:

$$h = 1 + \varepsilon h_1, \quad t_0 = t, \quad t_1 = \varepsilon t$$

from Eq. (1) obtain:

$$\frac{\partial h_1}{\partial t_0} + \frac{\mathrm{Re}}{Fr}(1 + Fr\tau_0)\frac{\partial h_1}{\partial x} = 0, \quad (2)$$

$$\frac{\partial h_1}{\partial t_1} + \frac{\mathrm{Re}}{Fr}(2 + Fr\tau_0)h_1\frac{\partial h_1}{\partial x} + \frac{W\mathrm{Re}\varepsilon}{3}\frac{\partial^4 h_1}{\partial x^4} + \frac{2}{15}\frac{\mathrm{Re}^3}{Fr^2}(1+Fr\tau_0)\frac{\partial^2 h_1}{\partial x^2} + \frac{1}{2}\mathrm{Re}\tau_0\int i\hat{h}_{1k}k^2\tau(k)e^{ikx}dk = 0 \quad (3)$$

From Eq. (2) it follows that in the first approximation (in a fast time scale) perturbations of small but finite amplitude propagate with a characteristic constant velocity: $c_0 = \mathrm{Re}(1/Fr + \tau_0)$. Eq. (3) describes the nonlinear evolution of perturbations on large (slow) times.

The characteristic longitudinal scale l_0 is chosen for coefficients of the second and fourth derivatives in Eq. (3) to be the same. Then, for ε we obtain:

$$\varepsilon = 0,4\mathrm{Re}^2(1 + Fr\tau_0)/\left(W Fr^2\right).$$

Considering this choice, after the replacement:

$$t = bt_1, \quad h_1 = AH, \quad b = W\mathrm{Re}\varepsilon/3, \quad A = 2Frb/\mathrm{Re}(2 - Fr\tau_0)$$

Eq. (3) takes the form [18]:

$$\frac{\partial H}{\partial t} + 2H\frac{\partial H}{\partial x} + \frac{\partial^2 H}{\partial x^2} + \frac{\partial^4 H}{\partial x^4} + B\int_{-\infty}^{\infty} ik^2\tau(k)\hat{H}(k,t)e^{ikx}dk = 0 \quad (4)$$

Here $B = \mathrm{Re}\tau_0/(2b) \equiv 3\tau_0/(2W\varepsilon)$.

Thus, in case of small Reynolds numbers, the problem of perturbations on the surface of the liquid film flowing under the action of gravity in the known stress field at the interface, is reduced to considering solutions of one nonlinear integro-differential equation. In the case of searching spatial periodic solutions, the integral term in (4) is transformed to the corresponding series.

The aim of this work is to find steady-state traveling periodic and soliton solutions of Eq. (4).

Eq. (4) is an interesting example of model equations, arising in the study of the perturbations evolution in active-dissipative media. The instability of linear perturbations is expressed by its terms with a second derivative and a term, containing the integral (the latter is due to accounting of friction disturbances at the film-gas interface), and dissipation is expressed by the fourth derivative, modeling capillary effects. The respective impacts of these terms are easily illustrated by the study of linear stability of the unperturbed solution $H = 0$. Indeed, neglecting the nonlinear term in (4) for its linear solutions $H \sim \exp(ik(x - ct))$ obtain the dispersion relation:

$$c \equiv c_r + ic_i = i\left(k - k^3\right) + Bk\tau(k) \quad (5)$$

Perturbations are unstable if the imaginary part of the phase velocity c is positive. Since the second term in the right part of Eq. (5) responsible for the stability of perturbations with decreasing wave number k decreases faster than, for example, the first one, it is clear that the unstable are the long-wavelength perturbations. Their wave numbers are smaller than the neutral wave number k_n, that satisfies the equation:

$$1 - k_n^2 + B\tau_{im}(k_n) = 0 \quad (6)$$

As is clear from (6), for the freely falling film ($B = 0$) the neutral wave number $k_n = 1$. We choose parameters of the unperturbed flow for the neutral wave number k_n to differ from unity. At that it is required for this value of k_n to correspond to a definite value of τ_{im}. The sought value of the parameter B is determined from (6):

$$B = \left(k_n^2 - 1\right)/\tau_{im}(k_n).$$

In the points with neutral wave numbers k_n the periodic steady-state traveling linear solutions branch from the trivial solution $H = 0$. As it is clear from (5), they have a phase velocity and frequency, respectively:

$$c_0 = Bk_n\tau_r(k_n), \quad \omega_0 = k_n c_0 = Bk_n^2(k_n)$$

The work [19] showed that in the vicinity of the neutral wave number k_n the steady-state traveling solutions of small but finite amplitude have the form:

$$H = A\exp\left[i(kx - \omega\tau)\right] + A^2 A_{H2}\exp\left[2i(kx - \omega\tau)\right] + C.C. \quad (7))$$

Here $k = k_n + A_k A^2$, $\omega = \omega_0 + A_\omega A^2$, C.C. is the complex-conjugated expression. Coefficients A_{H2}, A_k, A_ω depend only on $\tau(k_n)$, $d\tau(k)/dk|_{k_n}$, $\tau(2k_n)$. Due to the bulkiness their explicit form is not given here. The expression for the phase velocity with accuracy to A^2 has the form:

$$c \equiv \frac{\omega}{k} = \frac{\omega_0 + A_\omega A^2}{k_n + A_k A^2} = c_0 + \frac{A^2}{k_n}(A_\omega - c_0 A_k) \quad (8)$$

To determine the linear response of stresses to interface perturbations the authors of [2,18,20] used still popular linear models of turbulent gas flow over a wavy surface: the model of Benjamin (BM) [21] and boundary conditions transfer to the unperturbed level (BCT) [2]. So, for example, data obtained in [18] well agree both with calculated and experimental data on friction pulsations from [22,23]. There, using results of [2] the authors studied the stability of film flows based both on the integral model and Orr-Sommerfeld equations.

Obtaining results presented below, we used data on friction pulsations obtained in [18,20]. In calculations, the profile of the averaged gas flow velocity from [24] was used.