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Chaotic magnetic field lines and fractal structures in a tokamak with magnetic limiter

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ABSTRACT

Open hamiltonian systems have typically fractal structures underlying chaotic dynamics with a number of physical consequences on transport. We consider such fractal structures related to the formation of a chaotic magnetic field line region near the tokamak wall and the corresponding field line dynamics, described by a two-dimensional area-preserving map. We focus on the exit basins, which are sets of points which originate orbits escaping through some exit and are typically fractal, also exhibiting the so-called Wada property. We show qualitative as well as quantitative evidences of these properties.

1. Introduction

An outstanding problem in the physics of magnetically confined fusion plasmas is the control of the interaction between plasma particles and the inner wall in which they are supposed to be contained (e.g. in a tokamak) [1]. Due to the so-called anomalous transport, plasma particles escape towards the wall and are lost, reducing the plasma density and thus the quality of confinement [2,3]. Moreover, since many of these particles can be highly energetic they interact with the metallic wall causing the release of contaminants through sputtering and other processes [4]. One of the proposed ways to control plasma-wall interactions is the use of chaotic magnetic fields near the tokamak wall, spanning both the plasma outer edge and the scrape-off layer between the plasma and wall [5–9].

Such a chaotic field line region can be obtained by applying suitable magnetic perturbations that break the integrability of the magnetic field line flow, causing the appearance of area-filling stochastic lines (ergodic magnetic limiter) [10]. The initial claim was that such chaotic field would uniformize heat and particle loadings, so diminishing localized attacks and their undesirable consequences [5–7]. However, both theoretical and experimental evidences suggest that this is not true, i.e. the chaotic region can be highly non-uniform and thus the risk of localized loadings can only be diminished, if not completely avoided [11–16].

This non-uniformity of chaotic regions is ultimately the consequence of the underlying mathematical structure of chaotic orbits in area-preserving mappings, since they are structured on a complicated homoclinic tangle formed by intersections of invariant manifolds stemming from unstable periodic orbits embedded in the chaotic orbits [17,18]. Hence it is important to use such knowledge to understand how the escape is structured near the tokamak wall, locating possible "hot spots" (i.e. high loading parts of the wall) that should be protected so as not to cause extremely localized particle fluxes from the plasma, for example by using divertor configurations [19–21].

The main goal of this paper is to show the presence of fractal structures in the chaotic region of escaping magnetic field lines caused by an ergodic limiter, by using a simple area-preserving mapping proposed by Martin and Taylor [22]. We observe the presence of fractal basins, in the case of two exits [17], and the so-called Wada property in the case of three (or more) exits [23]. We emphasize the physical consequences of those mathematical properties in terms of escape times and magnetic footprints caused by the self-similarity of the fractal structures [24].

In previous papers dealing with exit basins due to ergodic limiter fields we have considered the fractality of exit basins from the direct calculation of the box counting dimension of the exit basin boundaries, and characterized the Wada property by a qualitative argument [11,12]. This may be an insufficient characterization of

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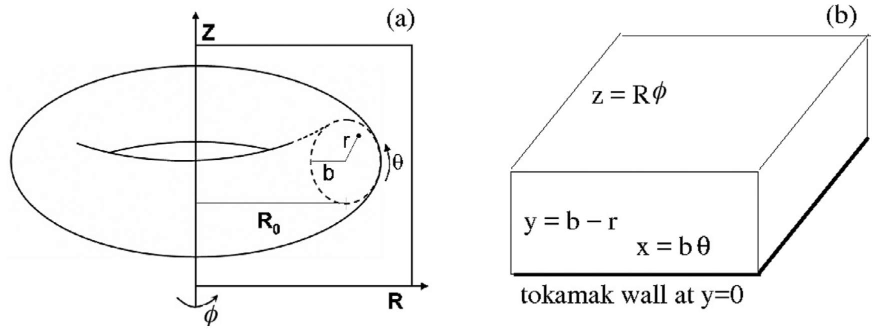

Fig. 1. Schematic figure showing the basic tokamak geometry in (a) local and (b) rectangular coordinates.

the exit basin structure because we would like to know the answer to questions like: (i) to which degree are the fractal basins mixed together; or (ii) to what extent is the Wada property fulfilled? In this paper we provide answers to these questions by developing further the analysis by presenting quantitative techniques introduced recently by Sanjuán and coworkers [25–27].

This paper is organized as follows: in Section 2 we outline the physical model used to describe the ergodic magnetic limiter as well as the mapping equations governing the dynamics of the corresponding magnetic field line flow. Section 3 considers the fractal nature of the escape basin boundary, both qualitatively (using the invariant manifold structure as a guide) and quantitatively (by computing the box-counting dimension of the boundary). In Section 4 we use the concepts of basin entropy and basin boundary entropy to characterize the fractality of the exit basins. Section 5 is devoted to a discussion of the Wada property for three exit basins and to the application of a technique (grid approach) to verify in what extent the Wada property is fulfilled by the system. The last Section is devoted to our Conclusions.

2. Magnetic field line map

A tokamak is a toroidal device for magnetic confinement of fusion plasmas, in which the plasma is generated by the ionization of an injected low-pressure gas (hydrogen) through an induced emf caused by the discharge of a capacitor bank [28]. There are two basic confining magnetic fields: a toroidal field $\mathbf{B}_T$ generated by coils mounted externally to the toroidal chamber and a poloidal field $\mathbf{B}_P$ generated by the plasma itself. In the axisymmetric case the resulting magnetic field lines are helicoidal, winding on nested toroidal surfaces (called magnetic surfaces) [29].

A general non-axisymmetric perturbation will break up a certain number of these magnetic surfaces. As a result, some of the magnetic field lines will no longer lie on surfaces, ergodically filling a bounded volume inside the toroidal device. Such magnetic field lines will be called chaotic but, since we are dealing with strictly time-independent configurations, the word chaos must be intended in a Lagrangian sense: two field lines, originated from very close points, separate spatially at an exponential rate as winding around the torus. Such description can be made rigorous by considering a hamiltonian description of magnetic field lines [10].

The basic tokamak geometry is depicted in Fig. 1(a) using cylindrical coordinates (R, ϕ, Z): the tokamak vessel is a torus of major radius $R = R_0$ with respect to the major axis parameterized by coordinate Z). The major radius stands for the minor axis, such that the position of a field line point can be assigned to the local coordinates (r, θ, ϕ), where $R = R_0 + r\cos\theta$ and $Z = r\sin\theta$. The variables θ and ϕ are also called poloidal and toroidal angles, respectively. The tokamak wall is located at $r = b$.

In this paper we will focus on the production of chaotic field lines near the wall of the toroidal device. Hence we can use a simplified rectangular geometry to describe the situation therein, so avoiding the use of unnecessarily complicated coordinate system. In this slab geometry description we use three coordinates (x, y, z) to describe a field line point, where $x = b\theta$ stands for the rectified arc along the wall, and $y = b - r$ is the radial distance measured from the wall (which is located at $y = 0$) [Fig. 1(b)]. The coordinate $z = R\phi$ measures the rectified arc along the toroidal direction.

The magnetic field line equations in these coordinates are written as

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} \tag{1}$$

where the equilibrium magnetic field is $\mathbf{B}_0 = \mathbf{B}_T + \mathbf{B}_P$. In the lowest-order approximation we set $\mathbf{B}_T = B_0\hat{\mathbf{e}}_z$ and the field line equations can be written as [30]

$$\frac{dx}{dz} = \frac{1}{2\pi R_0}[\alpha + sy + o(y^2)], \tag{2}$$

$$\frac{dy}{dz} = 0, \tag{3}$$

where the parameters α and s are related to the equilibrium magnetic field components, by expanding the safety factor $q(r)$ in a power series around the plasma boundary $r = b$, such that $\alpha = 2\pi b/q_b$ and $s = 2\pi bq'(b)/q_b^2$ [30]. Let us take, for example, a quadratic profile $q(r) = q_b r^2/b^2$. This yields $s = 4\pi/q_b$. Hence our choice $s = 2\pi$ would correspond to $q_b = 2$, which is a value in agreement with Tokamak experiments.

The equations (2) and (3) can be integrated, using z as the independent variable, such that a Poincaré map is obtained by sampling the values of x and y just after the nth crossing of a surface of section at $z = 0$ (the z-direction has a well-defined periodicity corresponding to the long way around the torus). In this way we define discrete-time variables (x_n, y_n) which are known as functions of the same variables at the previous crossing, namely (x_{n-1}, y_{n-1}).

The axisymmetric configuration (closed, toroidal magnetic surfaces) can be described in such a framework by a simple twist map (after a convenient rescaling of variables, such that $0 \le x < 2\pi$ and $y \ge 0$)

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \mathrm{T}_1 \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} x_n + s(y_n)\, y_n \\ y_n \end{pmatrix}, \tag{4}$$

The equality of y for consecutive iterations of the map reflects the fact that a field line always lies on a magnetic surface $y = const.$.

In this paper we consider a particular kind of symmetry-breaking perturbation on the above equilibrium configuration, the so-called ergodic limiter, which is a ring-shaped coil at $r = b$ with m pairs of straight wire segments of length ℓ, aligned with the toroidal direction and conducting a current I [Fig. 2]. The ergodic