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Mathematical modelling of physically/geometrically non-linear micro-shells with account of coupling of temperature and deformation fields

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ABSTRACT

A mathematical model of flexible physically non-linear micro-shells is presented in this paper, taking into account the coupling of temperature and deformation fields. The geometric non-linearity is introduced by means of the von Kármán shell theory and the shells are assumed to be shallow. The Kirchhoff-Love hypothesis is employed, whereas the physical non-linearity is yielded by the theory of plastic deformations. The coupling of fields is governed by the variational Biot principle. The derived partial differential equations are reduced to ordinary differential equations by means of both the finite difference method of the second order and the Faedo-Galerkin method. The Cauchy problem is solved with methods of the Runge-Kutta type, i.e. the Runge-Kutta methods of the 4th (RK4) and the 2nd (RK2) order, the Runge-Kutta-Fehlberg method of the 4th order (rkf45), the Cash-Karp method of the 4th order (RKCK), the Runge-Kutta-Dormand-Prince (RKDP) method of the 8th order (rk8pd), the implicit 2nd-order (rk2imp) and 4th-order (rk4imp) methods. Each of the employed approaches is investigated with respect to time and spatial coordinates. Analysis of stability and nature (type) of vibrations is carried out with the help of the Largest Lyapunov Exponent (LLE) using the Wolf, Rosenstein and Kantz methods as well as the modified method of neural networks.

The existence of a solution of the Faedo-Galerkin method for geometrically non-linear problems of thermoelasticity is formulated and proved.

A priori estimates of the convergence of the Faedo-Galerkin method are reported. Examples of calculation of vibrations and loss of stability of square shells are illustrated and discussed.

1. Introduction

In recent years, one of the most intensively and dynamically developing branches of industry is the micro-system technology, which includes miniature sensors of inertial and external information, micro-engines and transducers. Modern micro-mechanical and micro-electromagnetic systems include micro-mechanical gyroscopes (MMG) and accelerometers (MMA), which are better than traditional gyroscopes with rotating rotors in terms of mass-size characteristics, production costs and energy consumption parameters. This is why the MMG and MMA are widely employed in navigation industry, automotive industry, military equipment as well as in aircraft and rocket industries. For precise instrumentation, the problem of improving the accuracy of sensors plays a crucial role. Its solution consists in the application of new technological methods, in the construction of precise mathematical models of motion of sensitive elements as well as in the development of algorithms for minimisation of errors in the operation of devices. Nowadays, requirements put on the precision sensors of inertial information and conditions of their exploitation imply the need for deep investigations of the influence of various and qualitatively different types of physical factors and exciting processes on the sensor dynamics. The investigations include also the combined effect of the mechanical and thermal fields as well as a possibility of occurrence

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of the deterministic chaos in the micro-mechanical sensors of inertial information (MMDII).

Since the middle of the last century, the intensive development of thermoelasticity has been observed, which yielded success in numerous fields of technology such as the design of new constructions of steam and gas turbines, reaction and rocket engines, high-velocity airplanes, nuclear reactors, high-energy particles accelerators, geothermal engineering, micro-mechanical systems, and other.

Elements of the mentioned structures operate under conditions of non-stationary heating, which changes the physical-mechanical characteristics of the materials and generates gradients of temperature accompanied by non-homogeneous heat expansion of these elements' parts.

Stresses arising from the thermal and mechanical effects can initiate the formation of cracks leading to the destruction of a structure as a whole one. The tendency of a material to crush increases when thermal and force excitations appear. The repeated similar excitations can cause non-invertible damages.

The so far described phenomena indicate the need to develop new theories to describe the coupling of the temperature and deformation fields. The equations of the classical theory of thermoelasticity based on the Fourier law [1] do not contain any elastic terms associated with the occurrence of thermal effects. This drawback has been was removed by Biot [2], who introduced a theory of coupled thermoelasticity. The application of the theory of heat transfer based on the Fourier law is reported in numerous papers, and only a few recent representative examples are given below.

Bracco [3] described two models implemented in the Matlab/Simulink environment to study heat transfer in metal and insulating parts of steam boilers in the combined-cycle power plants based on the heat transfer Fourier equation.

Molina et al. [4] investigated a mathematical model of the temperature distribution of biological tissue during thermal ablation. In this study, three heat transfer equations, including the classical Fourier, hyperbolic and relativistic equations, were studied and their analytical solutions were derived. It was shown that the obtained solutions to these three equations coincide with each other on relatively large time intervals.

In the paper by Yang et al. [5], the Fourier law of 1D heat transfer equations in fractal media was investigated. Based on the Picard's method of successive approximations, the introduced approximate solution serves for the development of a 1D local fractal integral of Volterra's second kind equation obtained by a transformation of the Fourier flow equations in discontinuous media.

Krysko V.A. et al. [6] developed the mathematical model of coupling of temperature and deformation fields for flexible rectangular plates and shells subjected to impact transverse loads. It was illustrated that taking account of coupling of the temperature and deformation fields can lead to dynamical loss of stability.

Wang et al. [7] constructed a model of thermo-mass gas employing the thermo-mass theory. The process of heat transfer is treated as the flow of thermo-mass gas in a medium controlled by the thermo-mass pressure gradient (potential field). The obtained law degenerates either to the Fourier law when all thermal inertial effects are negligible or the heat transfer problem when the space-dependent inertial effects are very small.

Brischetto and Carrera [8,9] considered the fully coupled thermo-mechanical analysis of one-layer and multi-layer isotropic and composite plates and shells. The authors studied three important problems: (i) static analysis of plates and shells with imposed temperature on the external surfaces; (ii) static analysis of plate and shells subjected to a mechanical load, with the possibility of considering the temperature field effects; (iii) a free vibration problem with the evaluation of the temperature field effects.

In addition to the Fourier theory, there exist also other theories of coupled thermo-elasticity, including the Lord–Shulman (LS), Green–Lindsay (GL), Green–Naghdi (GN) and Chandrasekharaih theories, etc.

The so far carried out brief review of the state-of-the-art related to the considered problem yields a conclusion that in spite of a large number of publications on coupled problems of thermoelasticity of micro-shells studied with the use of different models and theories, there are no works aimed at investigating the non-linear dynamics of geometrically and physically non-linear and coupled thermoelastic problems.

The majority of the found researches are devoted to the analysis of problems dealing with a small number of degrees of freedom and solved by only one method. In fact, researchers do not solve the problem defined in the beginning. In order to obtain a true solution, one should solve the same problem with several alternative methods aimed at getting a solution to the system of an infinite number of degrees of freedom. In the present paper, reliability of the obtained results of the problem of vibrations of flexible and physically non-linear micro-shells is studied taking account of coupled temperature and deformation fields as well as the infinite dimension of the problem. This is realised by employment of two alternative methods of reducing a partial differential equation (PDE) to the Cauchy problem, i.e. the second-order finite difference method (FDM) and the Faedo-Galerkin method (FGM) in higher approximations. Convergence of these methods is also investigated by quantifying either a number of partitions of the integration domain (FDM) or a number of terms of the series composed of base functions Faedo-Galerkin method. The Cauchy problem is also solved by a few methods of the Runge-Kutta type. Determination of the sign of the Lyapunov exponents is carried out with the use of several different algorithms [10,11]. Besides, a theorem of existence of a solution to the problem of vibrations of flexible micro-shells is given taking into account the coupling of temperature and deformation fields. In addition, the a priori estimates of the FGM are given. Overall, it is claimed that the obtained solution is reliable/true with regard to the considered system with an infinite number of degrees of freedom.

In this work two types of the non-linearity are considered, i.e. geometric and physical. The physical non-linearity is based on the strain theory of plasticity. A solution to this issue can be found using the method of 'elastic solutions' [12]. Modification of this method to the method known as the 'method of variable elasticity parameters' has been introduced in reference [13], where the contact problem of interaction of the physically non-linear plates has been considered, i.e. both non-linearity effects, including the physical non-linearity and the design-type non-linearity occurred due to the contact/no-contact plates behaviour, have been addressed.

On each step of the used iteration procedure of the method of variable elasticity parameters, we have employed the method of variation iterations. The latter one reduces the problem of an infinite dimension governed by PDEs to that governed by ODEs. Convergence of the mentioned iteration procedure has been also proved.

2. Fundamental assumption and hypotheses

Let us consider a shallow rectangular shell with dimensions *a, b, h* along axes x_1, x_2, x_3, respectively. For a spherical shell, the internal radius, expressed in the shell thickness, can be easily determined with a formula $f = k_1/8$, where k_1 stands for the shell curvature parameter [14].

The origin of the coordinate system is located in the upper left corner of the shell, on its middle surface. The axes x_1, x_2 are parallel to the shell sides and the axis x_3 is directed towards the shell curvature (Fig. 1). In the given co-