



Chaos control of integer and fractional orders of chaotic Burke–Shaw system using time delayed feedback control



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ABSTRACT

The aim of this paper is to investigate the control of chaotic Burke–Shaw system using Pyragas method. This system is derived from Lorenz system which has several applications in physics and engineering (e.g. secure communications). The linear stability and the existence of Hopf bifurcation of this system are investigated. Based on the characteristic equation, a theorem is stated and proved. This theorem is used to calculate the interval values of the time delay τ at which this system is stable (unstable). By establishing appropriate time delay τ and feedback strength K ranges, one of the unstable equilibria of this system can be controlled to be stable.

We, also, introduced the fractional version of this system which is not studied in the literature as far as we know. The advantage of the fractional order system is that, the system has extra parameter which enriches its dynamics. Increasing the number of parameters may be used to increase the security of the transmitted information. We apply the Pyragas method to control the chaotic behavior of fractional Burke–Shaw system. As we did for the integer order, we determine the values of τ and K which guarantee that the fractional version is stable. Finally, to support the analytical results, some numerical simulations are carried out which indicate that chaotic solution is turned to be stable if τ passes through certain intervals. The bifurcation diagrams are calculated.

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1. Introduction

In the literature, due to unexpected behaviors that may emerge from a chaotic system, chaos control and synchronization have acquired increasing attention. Recently the trend of analyzing and understanding chaos has been extended to control and utilize chaos. The main goal of chaos control is to eliminate chaotic behavior and to stabilize the chaotic system at one of the system's equilibrium points. Many control techniques have been developed, such as passive control [1,2], optimal control method [3], impulsive control [4], and many others. Most of these methods are developments of two basic approaches: the Ott–Grebogi–Yorke method (OGY) [5] and time delay feedback control [6]. The main idea of the OGY control method is making only small time dependent perturbations of the parameter of chaotic systems since the system attractors have embedded within it an infinite number of unstable periodic orbits. The time delay feedback control method

was introduced by Pyragas [6]. The idea of this method is to inject an appropriate continuous controlling signal into the system which is proportional to the difference between the present state of a given system $Y(t)$ and its delayed value $Y(t - \tau)$. By choosing proper time delay, this difference is practically zero as the system evolves close to the desired steady state or periodic orbit which means stabilization. The advantage of this method is that the delayed control has no need for a reference system. Since it generates the control force from the information of the system itself. Moreover, this method is simpler and convenient in controlling chaos for a continuous dynamical system. Thus, time delayed feedback control has been used in a large variety of applications such as biology, medicine, chemistry, engineering and physics [7–10].

On the other hand, the theory of fractional calculus has become emerging field of research in both theories and applications in recent years. Now, fractional order differential equations have revealed to be significant mechanism in the modeling of numerous real-world applications of various fields such as physics, biology, engineering and so on [11–13]. The advantage of the fractional order system, is that, the system has extra parameter (or parameters) may be used to increase the security of the transmitted information [14,15]. The degree of freedom of a system is the number of

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parameters of the system that may vary independently. Therefore, some of the researchers have been introduced fractional calculus into neural networks and secure communication due to incorporate the infinite memory (previous response) [16,17]. There are different analytical techniques have been used to study the fractional order systems such as multiple-scale approach [18], the differential transform method [19], the homotopy analysis method [20] and averaging method [21–23]. Moreover, fractional order systems with time delay are commonly encountered in many fields. For example, the dynamic behavior of HIV infection of CD4⁺ T-cells and motion control systems with actuator limitation are introduced depending on delay fractional order [24–26]. Wen et al. studied the dynamical characteristics of Mathieu–Duffing oscillator under fractional order feedback coupling with time delay [27]. Designing circuit models for synchronization of fractional order chaotic systems with time delay are investigated by Atan [28]. There are a lot of control methods in fractional order dynamical systems such as active control [29,30], sliding mode control [31] and tracking control [32].

Now, owing to the significance of the earlier mentioned work related to feedback control method, we should record that many authors became interested in studying the effect of this method to control chaotic systems with integer order see [33–37]. To the best of our knowledge, although there are numerous results discussing the control of chaotic system with integer order using time delayed feedback control, few results on the literature discussing the control of chaotic systems with fractional order, such as Gjurchinovski et al. [38]. They showed that the time delayed feedback control can be used to stabilize unstable steady states and unstable periodic orbits in fractional-order Rössler system. While Huang et al. used delayed feedback controller to control the creation of bifurcation for delayed fractional predator–prey system with incommensurate orders [39].

The main contribution of this paper is to control Burke–Shaw chaotic system [40] via the time delayed feedback control. Regarding the time delay as a parameter, the effect of time delay on the dynamics of this system is explored. More specifically, we study the local stability and the existence of Hopf bifurcation of this system. In addition, we demonstrate that chaos vanishes as the time delay reaches a certain value and we try to extend these results for fractional version of this system.

The remainder of the paper is organized as follows: in Section 2, we give a brief description of Burke–Shaw system [40] with integer and fractional orders. In Section 3, we investigate the problem of controlling chaos of the Burke–Shaw system with integer order. We consider the stability of one of the fixed points and determine the ranges of delay τ at which the fixed point of the chaotic system can be controlled to be stable. We also obtain some sufficient conditions under which the Hopf bifurcations occur. While, in Section 4, we turn to scrutinize the control of fractional version of Burke–Shaw system via time delayed feedback control. We discuss the stability of the equilibrium and the occurrence of Hopf bifurcations. A bifurcation set is provided in the appropriate parameter plane. Finally, a conclusion is addressed to summary our works.

2. Description of Burke–Shaw system with integer and fractional order

The Burke–Shaw system has been derived by Burke and Shaw from the Lorenz equations [40]. It is a three dimensional system that possesses two quadratic nonlinear terms. The Burke–Shaw system was evolved from the Lorenz system, and therefore has similar algebraic structure to that of the Lorenz system, but is topologically nonequivalent. Detailed structural and parametric analyses have been reported in [41,42]. The governing equations of

the system are:

$$\begin{aligned}\dot{x}(t) &= -S(x(t) + y(t)), \\ \dot{y}(t) &= -y(t) - Sx(t)z(t), \\ \dot{z}(t) &= V + Sx(t)y(t),\end{aligned}\quad (1)$$

where x , y and z are the state variables, S and V are constant parameters. We note that system (1) is invariant under the change of variables $(x, y, z) \mapsto (-x, -y, z)$. Consequently, it is symmetric with respect to the z -axis (i.e. $(-x(t), -y(t), z(t))$ is also a solution). For $S > 0$, $V > 0$ this system has two fixed points:

$$F^{\pm} = \left(\pm \sqrt{\frac{V}{S}}, \mp \sqrt{\frac{V}{S}}, \frac{1}{S} \right).$$

The Burke–Shaw system (1) exhibits a chaotic attractor for the values $V = 4.272$ and $S \in [1, 12.06]$. Also, it has chaotic attractor when $S = 10$ and $V \in [1.64, 2.4] \cup [3.65, 20]$. In Fig. 1, we plot the Lyapunov exponents versus S and V . It is clear that one of them is positive which means that system (1) is chaotic.

We introduce the fractional version of Burke–Shaw system as:

$$\begin{aligned}D^{\alpha}x(t) &= -S(x(t) + y(t)), \\ D^{\alpha}y(t) &= -y(t) - Sx(t)z(t), \\ D^{\alpha}z(t) &= V + Sx(t)y(t),\end{aligned}\quad (2)$$

where $x, y, z \in \mathbb{R}$, $S, V > 0$, $\alpha \in (0, 1]$ and D^{α} is the Caputo differential operator of order α [43]. Systems (1) and (2) have the same equilibrium points. Assuming the Jacobian matrix of (2) at the equilibrium point F^+ is

$$J(F^+) = \begin{pmatrix} -S & -S & 0 \\ -1 & -1 & -\sqrt{VS} \\ -\sqrt{VS} & \sqrt{VS} & 0 \end{pmatrix}.$$

Let the eigenvalues of $J(F^+)$ be $(\lambda_1, \lambda_2, \lambda_3)$. According to the theory of stability for fractional order systems if $|\arg(\lambda_l)| > \frac{\alpha\pi}{2}$, ($l = 1, 2, 3$), then (2) is locally asymptotically stable at the equilibrium point. For the choice $S = 10$, $V = 4.272$ and $\alpha = 0.95$, the equilibrium points of (2) are

$$F^{\pm} = (\pm 0.6536, \mp 0.6536, 0.1).$$

The corresponding eigenvalues at F^+ are $\lambda_1 = -12.8499$, $\lambda_{2,3} = 0.9249 \pm 8.102i$ and that $\arg(\lambda_1) = \pi$, $\arg(\lambda_2) = 1.45712$ and $\arg(\lambda_3) = 4.82607$. Therefore, the equilibrium point F^+ of (2) is unstable.

3. Chaos control of integer order Burke–Shaw system (1)

In this section, we investigate the problem of controlling chaos of system (1). We add the control function $K(y(t) - y(t - \tau))$ to the second equation of system (1), where $\tau \geq 0$ is a constant time delay, K is the feedback strength. Then the delayed feedback control system can be written as:

$$\begin{aligned}\dot{x}(t) &= -S(x(t) + y(t)), \\ \dot{y}(t) &= -y(t) - Sx(t)z(t) + K(y(t) - y(t - \tau)), \\ \dot{z}(t) &= V + Sx(t)y(t).\end{aligned}\quad (3)$$

By using the delay τ as a bifurcation parameter in controlled system (3), we get the conditions under which chaotic attractor of this system vanishes. By linearizing system (3) at F^+ yields the following linear system:

$$\begin{aligned}\dot{x}(t) &= -S(x(t) + y(t)), \\ \dot{y}(t) &= -y(t) - Sz^*x(t) - Sx^*z(t) + K(y(t) - y(t - \tau)), \\ \dot{z}(t) &= V + Sy^*x(t) + Sx^*y(t).\end{aligned}\quad (4)$$

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