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Dynamical analysis of a reaction-diffusion phytoplankton-zooplankton system with delay

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ABSTRACT

A phytoplankton-zooplankton system with the delay and reaction-diffusion term is investigated. Firstly, existence and priori bound of solution without delay system are given. The stability of the axial steady state solution with delay system are analyzed by using the comparison arguments and modifying the coupled lower-upper solution pairs. By considering the effects of delay and diffusion, the stability and Hopf bifurcation of the positive steady state solution is investigated. When the delay does not exist, the diffusion cannot vary the stability of the steady state solutions, that is, the Turing instability cannot occur. When the delay exists, the effects of big and small diffusions to Hopf bifurcation are investigated, under certain conditions, the space inhomogeneous periodic solutions may produce. Furthermore, the algorithm determining the properties of bifurcation periodic solutions is given. At last, some numerical simulations are carried out to confirm the correctness of theoretical analyses.

1. Introduction

Plankton are at the bottom of most oceanic food webs, therefore, it is important to understand the dynamics of planktonic ecosystems. The first trophic level consists of phytoplankton which uptakes nutrient through photosynthesis. As we know, phytoplankton is not only the basis for all aquatic food chains, but also it can produce 70% atmospheric oxygen and absorb half of the carbon dioxide. Experiments and theoretical analyses have confirmed that phytoplankton has important effects between ecology and biogeochemical cycles [1]. The second trophic level is formed by zooplankton which feeds on phytoplankton and serves as a most favorable food source for fish and other aquatic animals. Modelling is a way to describe interactions inside an ecosystem, which can simulate ocean circulation to study its behavior in a pelagic environment. During the recent years, many authors have studied the system between phytoplankton and zooplankton [2–6].

A global increase in harmful plankton bloom has produced great socioeconomic damage in last two decades [7–9]. A harmful algal bloom (HAB) is an algal bloom that causes negative impacts to other organisms via production of natural toxins, mechanical damage, or by other means. There has been considerable scientific attention toward HAB and its control [10–13]. It is well known that HAB is often associated with large-scale marine mortality events and various types of shellfish poisonings. Therefore, the toxic substance produced by phytoplankton is an important aspect, which can reduce the grazing pressure of zooplankton. The impact of zooplankton grazing on HAB is also an important study task of the plankton ecology. If zooplankton community's grazing impact on initial stages of HAB is sufficiently high, then a bloom does not develop [14]. Authors in [4,11] study the optimal harvesting policy and the dynamical behavior of a toxin-producing phytoplankton-zooplankton system. In [10], the nutrient-plankton-zooplankton interaction models with a toxic substance which inhibits either the growth of phytoplankton, zooplankton or both trophic levels are proposed and studied. In [12], the authors construct a mathematical model for describing the interaction between a nontoxic and toxic phytoplankton with a single nutrient.

In addition, the liberation of toxic substances by phytoplankton species is not an instantaneous process but is mediated by some time lag requiring for maturity of the species. The zooplankton mortality due to the toxic phytoplankton occurs after some time lag of the bloom. The system with time lag leads to delay differential equation (DDE), which has been studied intensively and systematically [15–17]. The theory and application of DDE are emerging as an important area of investigation, since it is far richer than the corresponding theory of ordinary differential equation (ODE). For example, in [13], authors consider a mathematical model consisting of two harmful phytoplankton and zooplankton where the

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mortality of zooplankton due to the liberation of toxic substances by harmful phytoplankton occurs after an incubation time.

In this paper, we firstly structure a phytoplankton-zooplankton system with reaction-diffusion term and delay. In the system, the considered biological processes include: zooplankton prey phytoplankton for itself growth, phytoplankton follows Logistic growth and can produce toxin, the toxin can decrease the quantity of zooplankton, two populations may move along with currents. The theoretical approaches are based on reaction-diffusion equation theory and partial functional differential equation theory. The main studied problems include: existence and priori bound of solution without delay, stability and Hopf bifurcation of steady solution with delay by considering the effects of delay and diffusion. An algorithm is also given to investigate the direction and stability of the Hopf bifurcation. At last, a numerical example is given to support the theory analyses.

2. Methods

In this paper, the model considered bases on the following toxin phytoplankton zooplankton system [13,18,19]:

$$\begin{cases} \dfrac{dP(t)}{dt} = rP(t)\left[1-\dfrac{P(t)}{K}\right] - \beta H(P(t))Z(t), \\ \dfrac{dZ(t)}{dt} = \beta_1 H(P(t))Z(t) - \delta Z(t) - \rho T(P(t-\tau))Z(t), \end{cases} \tag{1}$$

where $P(t)$ and $Z(t)$ are the densities of the toxin-producing phytoplankton (TPP) population and zooplankton population at time t. The parameter r is the intrinsic growth rate and K is the environmental carrying capacity of TPP population. The term $H(P(t))$ describes the functional response for the grazing of phytoplankton by zooplankton, and the constant β is the maximum uptake rate for zooplankton species. β_1 denotes the ratio of biomass conversion (satisfying the restriction $0<\beta_1<\beta$). The parameter ρ is the release rate of toxic substances produced by per unit biomass of phytoplankton. The term $T(P(t-\tau))$ represents the distribution function of toxic substance which ultimately contributes to the death of zooplankton population. The zooplankton mortality due to the toxic phytoplankton is described by the term $\rho T(P(t-\tau))Z(t)$. The constant τ is the discrete time period required for the maturity of phytoplankton. δ is the natural death rate of zooplankton. All parameters are positive constants.

All of the above models are purely time dependent differential equations, that is to say, spatial effects are ignored. However, in the lakes and oceans it is clear that spatial effects will be very important. The individuals can go anywhere in the spatial domain since a well-known fact about Fickian diffusion leading to a Laplacian term. Plankton can move around because of diffusions and currents, and the effect of diffusion has been studied by authors [3,20–22]. The reaction-diffusion equation is the simplest equation with diffusion term [23–25]. In many biological and ecological systems, when the effects of spatial diffusion and delay are taken into account, parabolic functional differential equation (PFDE) is usually used to models [26–29]. Reaction-diffusion equation incorporating delay is more difficult to study although considerable progress has been made in recent years [30–34].

In this paper, a reaction-diffusion term is introduced into the system (1) and it assumes that the zooplankton reduces due to toxin with τ time lag and takes τ time to complete the process of converting phytoplankton to new cells, thus $e^{-\delta\tau}$ is the surviving rate of zooplankton. Moreover, we assume that the water body is closed, with no plankton species entering and leaving at the boundary. Considering spatial changes in both species, the system (1) changes the following reaction-diffusion system:

$$\begin{cases} \dfrac{\partial P(x,t)}{\partial t} = D_1\Delta P(x,t) + rP(x,t)\left[1-\dfrac{P(x,t)}{K}\right] \\ \qquad - \dfrac{\beta P(x,t)Z(x,t)}{1+\gamma_1 P(x,t)}, & x\in(0,l\pi),\ t>0, \\ \dfrac{\partial Z(x,t)}{\partial t} = D_2\Delta Z(x,t) \\ \qquad + \dfrac{e^{-\delta\tau}\beta_1 P(x,t-\tau)Z(x,t-\tau)}{1+\gamma_1 P(x,t-\tau)} \\ \qquad - \dfrac{e^{-\delta\tau}\rho P(x,t-\tau)Z(x,t-\tau)}{1+\gamma_2 P(x,t-\tau)} - \delta Z(x,t), & x\in(0,l\pi),\ t>0, \\ \dfrac{\partial P(x,t)}{\partial x} = \dfrac{\partial Z(x,t)}{\partial x} = 0, & x=0,\ l\pi,\ t\ge 0, \\ P(x,t) = \varphi_1(x,t)\ge 0, \quad Z(x,t) = \varphi_2(x,t)\ge 0, & x\in[0,l\pi], t\in[-\tau,0], \end{cases} \tag{2}$$

where $P(x, t)$ and $Z(x, t)$ denote the densities of phytoplankton population and zooplankton population at space x and time t, respectively. D_1 and D_2 denote the diffusion coefficients of phytoplankton and zooplankton. $l\pi$ denotes the depth of the water body. $1/\gamma_1$ and $1/\gamma_2$ are the half-saturation constants. The other parameters are the same senses as the system (1). The homogeneous Neumann boundary condition means that no plankton species is entering or leaving the water body at the top or bottom, $\Delta = \frac{\partial^2}{\partial x^2}$ is Laplacian operator. $(\varphi_1, \varphi_2) \in C := C([-\tau, 0], X)$, and X is defined by

$$X = \left\{ (P,Z) : P, Z \in W^{2,2}(0, l\pi),\ \left.\frac{\partial P}{\partial x}\right|_{x=0,l\pi} = 0,\ \left.\frac{\partial Z}{\partial x}\right|_{x=0,l\pi} = 0 \right\}$$

with the inner product $<\cdot,\ \cdot>$.

In order to reduce the number of parameters, introducing new variables

$$\hat{t} = rt,\ \hat{P} = \frac{P}{K},\ \hat{Z} = \frac{\beta Z}{r},\ \hat{\tau} = r\tau,\ \hat{\beta}_1 = \frac{K\beta_1}{r},\ \hat{\delta} = \frac{\delta}{r},$$

$$\hat{\rho} = \frac{\rho}{r},\ \hat{D}_1 = \frac{D_1}{r},\ \hat{D}_2 = \frac{D_2}{r},\ \hat{\gamma}_1 = \frac{\gamma_1}{r},\ \hat{\gamma}_2 = \frac{\gamma_1}{r},$$

and dropping hats, the system (2) can be rewritten as

$$\begin{cases} \dfrac{\partial P(x,t)}{\partial t} = D_1\Delta P(x,t) + P(x,t)[1-P(x,t)] \\ \qquad - \dfrac{P(x,t)Z(x,t)}{1+\gamma_1 P(x,t)}, & x\in(0,l\pi), t>0, \\ \dfrac{\partial Z(x,t)}{\partial t} = D_2\Delta Z(x,t) + \dfrac{e^{-\delta\tau}\beta_1 P(x,t-\tau)Z(x,t-\tau)}{1+\gamma_1 P(x,t-\tau)} \\ \qquad - \dfrac{e^{-\delta\tau}\rho P(x,t-\tau)Z(x,t-\tau)}{1+\gamma_2 P(x,t-\tau)} - \delta Z(x,t), & x\in(0,l\pi),\ t>0, \\ \dfrac{\partial P(x,t)}{\partial x} = \dfrac{\partial Z(x,t)}{\partial x} = 0, & x=0,\ l\pi,\ t\ge 0, \\ P(x,t) = \varphi_1(x,t)\ge 0,\ Z(x,t) = \varphi_2(x,t)\ge 0, & x\in[0,l\pi],\ t\in[-\tau,0]. \end{cases} \tag{3}$$

In the following, it will investigate the dynamic behaviors of the system (3) in details by considering the effects of diffusion and delay.

3. Existence and priori bound of solution for the system (3) with $\tau = 0$

In this section, a sufficient condition is given for the existence of a positive solution of the system (3) without delay, and a priori bound of solution is also given.