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Study on the effect of environmental pollution based on a fractional derivative resource depletion model

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ABSTRACT

The effect of resource usage on economic growth has been investigated in many papers and the generic effect of resource usage efficacy by improving technical skills has been discussed. However, the impact of environmental pollution and the effect of memory property of the economic variables on economic growth have not been studied in detail. In this paper, a new resource depletion model, in which fractional derivative describes the memory property of economic variables, is built. Firstly, the influence of the environmental pollution on economic growth is analyzed. The results show the model is extreme sensitive to some parameters with considering the impact of environmental pollution. The small change of the parameters can lead a collapse of the economy in a short time. Meanwhile, new equilibrium region of economic growth can be obtained with considering the effect of the memory property of economic variables. Secondly, the effect of externally stochastic perturbation on the average of capital is studied, and the result shows random excitation can not only impede but also promote the development of the economy.

1. Introduction

The study of economic growth is always one of the most important issues in Macroeconomic. Economic growth brings the increase of capital. High level of capital stands for the high average per capita income and better social welfare, which is the purpose of economy development. Economic growth is subjected to the technology, resource and the environmental carrying capacity. One often obtains the idea that technological improvements allow a better use of resources and more substitution among different resources [1–4], which is encountered by many economists. However, the improvements of technology also bring more consumption of resources. The alternative view is that technological process accelerates resource depletion. From 1750 to 21st century, the average wealth has a staggering growth and the growth rate is becoming more faster. But the improvement of wealth is at the cost of huge amount of resource depletion including nonrenewable resources. The research of the relation between technological improvements and resource depletion with economic growth is the way to get the answer. The process of capital accumulation is the process of resource depletion. With the economic growth and more resource depletion, serious damage to the environment is also brought [5].

The feedback loops of technological improvement and resource depletion has been extensively studied [6–8] and many models have been built [8–10]. But only few papers consider the effect of environmental pollution on economic growth. Bruno Nkuiya [11] investigated a dynamic game among countries that face the fact that complex systems exposed to pollution may suddenly and permanently shift to a dangerous regime. Wang et al. [12] study a dynamic overlapping-generations model where pollution worsens risk of future ill health.

When analyzing the time series, the series often present autocorrelation. How to describe this property in a continuous model is undefined. T. Sccot [13] applied a memory function to depict this property when building the model to study the Hyperbolic memory discounting and the political business cycle. But this method is just an ideal assumption, and it can't correctly describe the memory property of variables. Fractional order derivative is shown to be suitable to describe the long memory property of the variables. Gemant [14] had a pioneering work in this field and he firstly used fractional calculus to investigate the equations of material behavior. Then, Caputo and Mainardi [15] used fractional calculus to compute the equations of viscoelastic materials and the computing results could be in accordance with the experimental results.

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Afterwards, many scholars gave deep researches in fractional calculus [16–22]. In recent years, a lot of papers of fractional modeling are published. Nick Laskin [23] investigated the equation of financial assets with fractional derivative and studied the probability distribution function (pdf) of the returns. Yin [24] designed a sliding mode control law to control chaos in a class of fractional order chaotic systems. And the fractional-order financial system is investigated by many scholars [25–32]. Huo et al [33] proposed a homogeneous-mixing population fractional model for human immunodeficiency virus (HIV) transmission, which incorporates anti-HIV preventive vaccines. As is known to us, capital of this year is affected by the level of capital years ago. If we have a large number of capitals last year, we have to invest more to fulfill the running of the capital. For the same reason, technological improvement, resource depletion, and environmental pollution all present this memory property. Thus, fractional order derivative can be integrated into the model to describe the memory property of these variables.

In the real world, externally stochastic perturbations are omnipresent and inevitable. The random factors are such as natural disaster, climate change, technology's development and even the behaviors of human beings. The dynamic characteristics can be more complicated when the random excitation and memory property are considered simultaneously. These externally stochastic perturbations can also influence the running of the dynamical economy system. The researches of stochastic responses of stochastic system with fractional derivative are studied by many scholars [34–37]. But, in the high dimension of fractional-order model is still to be investigated.

In this paper, the effect of pollution on the economic growth is discussed. And some surprising conclusions are obtained. This paper is organized as follows. In Section 1, model description is presented. Considering the memory property and environmental pollution, a fractional-order derivative model is built. In Section 2, local stability of the system with fractional derivative is investigated comparing with the classical model with integer order derivative. In Section 3, bifurcation properties of the parameters and the practical significance of the bifurcation in the model are studied. In Section 4, the effect of the externally stochastic perturbations on the level of capital is researched.

2. Model description

Capital represents the level of economic development. In order to study the interactions among technical skills, resources consumption and environmental pollution, we improve the model presented by Hilla Behar et al. [38], in which the environmental pollution and memory property of variables are not considered. The improved model can be written as follows:

$$\begin{aligned}
\frac{d^{\alpha_1}A}{dt^{\alpha_1}} &= \beta_A - \delta_A^1 A + \alpha_A B - \delta_A^2 P \\
\frac{d^{\alpha_2}B}{dt^{\alpha_2}} &= \beta_B ABC - \delta_B B \\
\frac{d^{\alpha_3}C}{dt^{\alpha_3}} &= \beta_C - k\beta_B ABC - \delta_C C \\
\frac{d^{\alpha_4}P}{dt^{\alpha_4}} &= \beta_P AB - \delta_P P
\end{aligned} \tag{1}$$

where A denotes technical skill, B denotes capital, C denotes resources, P denotes environmental pollution, α_i denotes fractional derivative order.

For fractional derivative [39], the Riemann-Liouville definition is chosen in this investigation, which can be written as

$$D^{\alpha_i}x(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{x(\tau)}{(t-\tau)^{\alpha_i}}d\tau, 0 < \alpha_i < 1 \tag{2}$$

The schematic interactions [38]:

(a) Knowledge has a natural creation and a natural decay. Investment of capital increases the technical skill and environmental pollution decreases the technical skill [12].
(b) Capital indicates the goods produced. So, it consumes the existing capital, technical skill and resources. And it has a natural decay.
(c) Resources have a natural creation and a natural decay. And the capital is the production of the resources.
(d) Environmental pollution has a natural decay. When we have a high level technical skill and capital, it has more contamination to be produced.
(e) Considering the memory property explained in "Introduction", the fractional order derivative substituted the integer order derivative. And different variables would have different memory properties.
(f) β_A denotes the natural creation rate of technical skill; A decays with a rate of δ_A^1; A is created by B with a rate of α_A; P has a negative effect on technical skill (the coefficient is δ_A^2).
(g) B is created by A, B, C with a rate of β_B, B decays with a rate of δ_A^2.
(h) C is created with a natural rete of β_C and is used by A, B, C with a rate of $k\beta_B$; C decays with a rate of δ_C
(i) P is created by A, B with a rate of β_P, and the nature has a self-purification ability, the purification rate is δ_P.

3. Equilibrium and local stability

The equilibrium state, corresponding to the fixed point of a dynamical system, is important to be concerned by economists. So the investigation of the stability of the fixed point has a practical significant. After considering the memory property and environmental pollution, one can obtain some nontrivial conclusions.

One can obtain the possible fixed points of the ODE model with four variables by computing the four equations of system (1).

In Eq. (1), one can obtain three possible non-negative fixed points E_1, E_2 and E_3

$$A^* = \frac{\beta_A}{\delta_A^1}, B^* = 0, C^* = \frac{\beta_C}{\delta_C}, P^* = 0, \quad (E_1)$$

$$A^* = \frac{\delta_B\delta_C}{\beta_B\beta_C - \beta_B\delta_B k B^*}$$

$$B^* = \frac{(\beta_B\beta_C\alpha_A\delta_P - \beta_A\beta_B k\delta_P\delta_B - \delta_B\delta_C\delta_A^2\beta_P) - \sqrt{\left(\beta_B\beta_C\alpha_A\delta_P - \beta_A\beta_B k\delta_P\delta_B - \delta_B\delta_C\delta_A^2\beta_P\right)^2 - 4\delta_P\delta_B\beta_B\alpha_A k(\delta_A^1\delta_B\delta_C\delta_P - \beta_A\beta_B\beta_C\delta_P)}}{2\delta_P\delta_B\beta_B\alpha_A k}$$

$$C^* = \frac{\delta_B}{\beta_B A^*}$$

$$P^* = \frac{\beta_P A^* B^*}{\delta_P}, \quad (E_2)$$