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# “Viral” Turing Machines, computation from noise and combinatorial hierarchies

Theophanes E. Raptis [a,b,*]

[a] Computational Applications Group, Division of Applied Technologies, National Center for Science and Research “Demokritos”, Athens, Attiki 15310, Greece
[b] Physical Chemistry Lab., National Kapodistrian University of Athens, Athens, Greece

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ABSTRACT

The interactive computation paradigm is reviewed and a particular example is extended to form the stochastic analog of a computational process via a transcription of a minimal Turing Machine into an equivalent asynchronous Cellular Automaton with an exponential waiting times distribution of effective transitions. Furthermore, a special toolbox for analytic derivation of recursive relations of important statistical and other quantities is introduced in the form of an Inductive Combinatorial Hierarchy.



## 1. Introduction

The last three decades, a new computational paradigm known as the interaction computational paradigm (ICP) [1–4] has slowly arisen out of the birth of the internet at the last two decades of 20th century. This has a lot to offer in the case of interacting systems in physics, biology and elsewhere from a purely informational viewpoint wherever a degree of information processing inside individual components in a network is present. Moreover, such informational storage and processing capacity might also prove possible even at a fundamental level of biochemistry [5,6]. Applications could range from biochemistry to the new field of soft robotics and nano-robotics.

Interactivity is ubiquitous in very many areas of complex systems dynamics. Stephen in [7] as well as Stephen and Dixon in [8] provide strong evidence towards fractal scaling in anticipatory behavior associated with waiting times in the context of organism – environment interaction. Grigolini also presents in [9] a thorough discussion on the biological emergence issue as a reason for moving beyond mere reductionism based on arguments from renewal processes [10] and anomalous diffusion [11] indicating mechanisms for breakdown of ergodicity. Last year, Hu et al. [12] also reported similar effects for single protein molecules in realistic, long in vivo times out of their intrinsically fractal energy landscape.

In the case of the ICP, original work by Wegner as well as more recently by Goldin on multi-tape persistent TMs, present a mathematical model of computation in the case of interruptive computation, intermittently forced to receive external inputs and/or provide appropriate outputs on a network or inside an arbitrary noisy environment. Wegner has even proposed that the ICP represents an example of “*Hypercomputation*” or “*Super-Turing Computation*” that moves beyond the confines of the so called “Turing Tarpit” or the associated but as yet formally unproved “Church-Turing Thesis” [13] claiming that in general, ICP contains certain non-algorithmic aspects. This particular issue proved quite thorny and it soon became a matter of fierce debate that strongly polarized the computer science community as can be seen in Davis [14,15] as well as in the more recent rebuttal in [16]. Recently, Dodig-Crnkovic suggested an alternate epistemological interpretation of information semantics as ICP in [17], associated with info-computationalism [18].

For the aims of the present report we can borrow the simplest original example by Wegner, known as the “Interactive Identity Machine” (IIM) as presented in [1]. The IIM model is a simple reflective mechanism or transducer such that given an indicator function over a set as a Boolean filter, and an iterator construct (*while…do*) it reads an input from some other assumingly intelligent, environmental resource and passes it to its output thus “mimicking” another agent by borrowing its own intelligent responses. While simplistic in its appearance, this trick allows more complex behaviors when put in some network or game with other intelligent agents. The second important aspect of IPC regarding the distinction of an internal discrete time in comparison with an

* Correspondence to: Computational Applications Group, Division of Applied Technologies, National Center for Science and Research “Demokritos”, Athens, Attiki 15310, Greece.

*E-mail address:* rtheo@dat.demokritos.gr

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external continuous time flow is also important for the type of stochastic point processes that will be examined here.

One may already notice a certain caveat in the original non-algorithmic argument from a strictly physical viewpoint in that no one has proven as yet the external environment to be really non-algorithmic or the opposite as in the case of so called, "Physical CTT" [19] (Pan-computationalism) identifying natural and algorithmic processes. However, we intend to show with a modification of the simple Wegner's example that under certain assumptions, pure noise of arbitrary statistics can be considered as a superposition of an at least countable infinity of arbitrary computations that can be filtered out.

To this aim, we will introduce in the next section a model of an Interactive "Viral" Turing Machine (IVTM) which is more complex than the original IIM yet simple enough for a toy model. In Section 2, we examine a transcription protocol for the complete arithmetization of the dynamics of such machines and in Section 3 we examine the resulting dynamics when interacting with a stochastic environment. In Section 4, we expand the arithmetization program towards a more complete treatment of generic automata and string rewriting machines by introducing the notion of combinatorial hierarchies. In the last section we also discuss the possible significance of a network of IVTMs interacting both with themselves and the environment in association with the problems of biological emergence and abiogenesis.

## 2. IVTMs as passive dynamical systems

It is well known that the older living organisms from the general class of virii, carrying only genetic material within an external protective capsule, are more or less in an inert condition like a "living-dead" piece of matter until their proximity to a host allows a phase transition due to the host's environmental temperature increase [20]. Then, the closely packed genetic material falls into a more liquid state allowing it to pass into the host's main body through pores using generic diffusion mechanisms.

In a similar sense we can define an abstract machine model from the original TM definition [21] by an appropriate dissection of certain elements requisite in their original definition. Any such TM requires (a) a set of internal control symbols, (b) a set of tape (memory) symbols a subset of which may stand for input and output symbols, (c) a permanent storage medium as a one dimensional memory, originally called the "tape" and (d) a left or right moving "head" upon the tape in order to read or write symbols. The head may also be of sufficient internal complexity to hold the internal control states as a primitive form of "*if…else*" statements. Given an alphabet for (a) and (b), an arbitrary TM is defined by an appropriate Transition Table for all permissible input and output states.

In the standard TM paradigm, an internal mechanism with a source of energy is assumed to guarantee the uninterrupted motion of the reading/writing head. In the IVTM instead, we shall assume a simplified version merging each tape position with additional symbols for a static version with no head making our reduced inert TM unable to perform iterations on its own. It will be only possible to do so when in touch with an external noise source like environmental heat. It is this pairing of an inert TM and a noise source that shall form a complete Interactive Viral TM (IVTM). In order to simplify the presentation, we shall choose as our toy model the recently introduced Wolfram's minimal (2,3) TM [22] which has also been realized recently as an artificial muscle machine [23].

To complete the required formal redefinition we shall have to translate every aspect of the original IVTM into a special version of *Asynchronous Cellular Automata* (ACA). We first define the IVTM total transition map via an additional technique which is often called

**Table 1**
Arithmetization of the Transition Table for the Wolframm (2,3) minimal UTM. Asterisks have been put in the indifferent states of the total of $2^4$ lexicographically ordered input states while the motion bit has been added on top.

| | | | | | |
|---|---|---|---|---|---|
| (0, 1) | *000 | (L/R)*** | *** | *000 | (0, 1) |
| (2, 3) | *100 | (L/R)A0 | RB1 | 1011 | 13 |
| (4, 5) | *010 | (L/R)B0 | LC0 | 0110 | 6 |
| (6, 7) | *110 | (L/R)C0 | LB0 | 0010 | 4 |
| (8, 9) | *001 | (L/R)*** | *** | *001 | (8, 9) |
| (10, 11) | *101 | (L/R)A1 | LC0 | 0110 | 6 |
| (12, 13) | *011 | (L/R)B1 | RC1 | 1111 | 15 |
| (14, 15) | *111 | (L/R)C1 | RA0 | 1100 | 3 |

"*Gödelianization*" or "*Arithmetization*" turning the original table into a list of integers by a change of encoding. The terminology originates in the older Goedel encodings [24,25] introduced for purposes of theoretical examination of logical proofs and first order logic. Practically, we are interested in all possible ways by which a tuple of $n$ integers can be mapped 1-1 in $N$ as a single integer. One can discern between three main cases

- Unbounded Tuples – Gödel code: for any arbitrary set $\{n_1, \ldots, n_k\}$ with undefined upper bound the fundamental theorem of arithmetic allows to always finding a unique integer $n$ given a list of $k$ distinct primes as $n = \pi_1^{n_1} \ldots \pi_k^{n_k}$.
- Bounded Tuples-maximal element code: for any set of integers with a given maximal element $n_{\max}$ as an upper bound for all sets, the maximal element is chosen as the basis $b = n_{\max}$ of a new alphabet such that by the polynomial representation there is always a unique integer $n = n_1 + n_2 b + \ldots + n_k b^{k-1}$.
- Bounded Tuples-maximal bit code: for any set of integers with a maximal element $n_{\max}$ of which the binary logarithm is defined as $l(n_{\max}) = [\log_2(n_{\max})] + 1$ there is always a unique integer such that $n = n_1 + n_2 2^l + \ldots + n_k 2^{kl-1}$.

It is the last (max. bit) encoding which is the most economical to use for practical purposes as for the arithmetization of TMs. The (2,3)-TM in particular uses a ternary alphabet for tape symbols and a binary alphabet for internal control states plus a motion bit for reading the next memory position via a pointer increase or decrease as shown in Table 1. This allows rewriting each column of the transition table as a set of at most 4-bit integers. For this it is also necessary to introduce an additional bit on top of the first column to restore the symmetry of a 4-bit to 4-bit integer transition. This can be used to represent the previous state of the motion bit which remains indifferent during computation. It should also be noticed that the particular tabulation is not unique and for non ordered tuples of $k$ integers one can find at least $k!$ ways of assignment to a single integer encoding, yet there is sufficient reason for the particular choice presented here in that it simplifies the expression of a set of evolution equations in the following paragraphs.

Unfolding the first column of Table 1 for the additional bit leads to a complete representation of all possible 16 states forcing also the addition of 4 idle states as fixed points during which no action is taken. The resulting integer sequence is the IVTM's original "genome" under an interpretation protocol that gives different meaning to parts of the binary decoding of each state value. Regarding the dynamics of the integer map given in Table 1, (Fig. 1) one observes that transitions are disjoint from the unreachable set of the four idle states (fixed points) which are unusable as memory states and stand for "null" or "vacuum" states. This is a peculiarity of the particular minimal implementation of Wolfram's TM which has no "halting" state. The particular subset can be omitted by assuming initial sequences from a hexadecimal alphabet only in the remaining set of 12 usable values. Any trajectory starting from this