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New studies for general fractional financial models of awareness and trial advertising decisions

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ABSTRACT

In this paper, two numerical techniques are introduced to study numerically the general fractional advertising model. This system describes the flux of the consumers from unaware individuals group to aware or purchased group. The first technique is an asymptotically stable difference scheme, which was structured depending on the nonstandard finite difference method. This scheme preserves the properties of the solutions of the model problem as the positivity and the boundedness. The second technique is the Jacobi–Gauss–Lobatto spectral collocation method which is exponentially accurate. By means of this approach, such problem is reduced to solve a system of nonlinear algebraic equations and are greatly simplified the problem. Numerical comparisons to test the behavior of the used techniques are run out. We conclude from the computational work that: the Jacobi–Gauss–Lobatto spectral collocation method is more accurate whereas the nonstandard finite difference method requires less computational time.

1. Introduction

It is well known that the aim of the advertising is to convince the consumers to purchase the products, that depending on the highlighting the neediness of the products in general and by showing the differentiation a specific brand over other products to encourage consumers to buy it.

There are several techniques to change audience opinion on products or services. One of these methods is advertising messages. These messages can be via body media as television, radio, newspapers and magazines, also these messages can be via soft media such as text messages, websites, etc. [1].

Studying of the advertisement strategies are very important with a view to increase the sales and to get better the company's earning. So, it is very useful to construct and study a proper dynamic advertisement model to describe the sales that depend on the time and on the audience population [2]. There are a lot of suggested models to describe the advertisements affair that set these problems from the point of view of marketing, economic, and operations management [1,3], where analyzing the advertising policies is done over the time using dynamical models [4,5]. These dynamical models are described by differential equations, where the market share, sales, subpopulations and all the critical state variables are assumed to be changed in continuous form with respect to the time. The purposes of each advertising are totally different. For instance, some of them are aim to compare between two, or among three or more trademarks. Another one is to introduce a new product to the market. Depending on these purposes the advertising models will be constructed.

Usually, taking the advertisement its effect is always delayed in time, thus incorporating the memory to differential models of advertising is needed. Therefore the models in which the current state depends on all of its previous states not only upon its first previous one are more suitable to describe the strategies of the advertising. It is well known that the fractional derivatives are defined in terms of an integral form over all history of the plan [6], therefore the derivatives models that described using fractional derivative is more suitable for the advertising problem. For many applications which involve epidemic models, the models that constructed depending on fractional order derivatives have been shown to output a better fit real data results than the models that constructed depending on integer order derivative [7].

Recently, fractional calculus has gained considerable popularity and importance due to its engaging applications as a novel modeling act in a variety of engineering and scientific fields, such as viscoelasticity [8], thermoelasticity [9,10], system control [6,11,12]

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hydrology [13] and fractional dynamics [14]. Fractional differential equations is the better way to describe the fractional models. Unfortunately, only in simple cases one can obtain the exact solutions of such differential equations [15]. Hence, it is necessary to solve these models numerically, which requires great effort. Some of the introduced numerical techniques for approximating the solutions of such problems are: finite element methods [16], finite difference methods [17–22], Adomian's decomposition method [23], homotopy perturbation methods [24], Taylor collocation methods [25], spectral methods [26,27], variational iteration methods [28] and higher order numerical methods [29].

On the other hand, the nonstandard finite difference method (NSFDM) is proposed by Mickens [30–34] for improving special discretizations of some terms in the differential equations, such that depending on the denominator function and the specific discretization this method be more accurate and more stable than standard method [35,36], in addition this method can be easy to formulate [37]. The positive applications of the NSFDM can be found in the fields of physics, chemistry, engineering [21,38,39] and [40]. Especially, the most attractive applications are in mathematical biology and ecology [41,42] such that the merit of the NSFDM has been shown prominently. In addition, the dynamic preserving properties of the NSFDM are also well performed in solving fractional-order system, such as the fractional-order neuron system [43], the fractional-order Rössler system [44], and the fractional Hodgkin–Huxley model [45].

Also, spectral methods are a very important group of methods in which the approximation solutions are expressed in terms of specific basis functions (see, e.g. [46–48] and the references therein). These basis usually are orthogonal polynomials [49–52]. Spectral methods have three well-known versions, collocation, galerkin and tau methods. The spectral collocation method has an exponential convergence rate, therefore it give highly accurate approximation solutions for nonlinear differential equations even the number of grid point is small. This methods was very popular to solve time-dependent differential equations. Choicing of collocation (grid) points are necessary to achieve the stability and convergence of the spectral collocation method [47]. The collocation method has been used for solving numerically both fractional order partial differential equations [26,27,53,54] and for fractional order ordinary differential equations. Recently it has been used for solving systems of fractional differential equations [55].

The main goal of this manuscript is to introduce comparative study between NSFDM and the Jacobi–Gauss–Lobatto spectral collocation method (JGLSCM) for solving numerically the following advertising fractional model of nonlinear differential equations:

$$\begin{aligned} {}_0^cD_t^\alpha x(t) &= -u^\alpha x(t) - \frac{k^\alpha}{N(t)}x(t)(N(t)-x(t)) + \mu_b^\alpha N(t) \\ &\quad - \mu_d^\alpha x(t), \\ {}_0^cD_t^\alpha y(t) &= u^\alpha x(t) + \frac{k^\alpha}{N(t)}x(t)(N(t)-x(t)) - (a^\alpha + \nu^\alpha)y(t) \\ &\quad + \delta^\alpha z(t) - \mu_d^\alpha y(t), \\ {}_0^cD_t^\alpha z(t) &= (a^\alpha + \nu^\alpha)y(t) - \delta^\alpha z(t) - \mu_d^\alpha z(t). \end{aligned} \tag{1}$$

All variables and parameters in the above system and their definitions are given in Table 1. It is important to notice that all the parameters here are depend on the fractional order α, and for simplicity in the notation in the sequence of this paper we will omit the letter α from the above of the parameters.

System (1) describes the flows of consumers from and into the different groups. Such that the total number $ux(t)$ of individuals transfer to the aware group $y(t)$ via advertising. In addition the consumers who know the product, i.e., $(N(t)-x(t))$, contact and inform a total of $k(N(t)-x(t))$, out of which only a fraction of $x(t)/N(t)$ are newly informed. This system was introduced by Muller [4] in case $\alpha = 1$ and developed by Chen-Charpentier *et al.* [2] to the fractional case. We interest in this fractional order case since the effect of advertising is not instantaneous, so incorporating the memory is very important to explain and understand advertising with two components: awareness and trial advertising. For more details about the description of this system we refer to [2,4,5].

The outlines of the article are as the following: In Section 2, we recall some relevant definitions on fractional calculus and introduce the preliminaries of NSFDM, also we give some useful properties of Jacobi polynomials. In Section 3, we construct nonstandard finite difference scheme (NSFDS) for system (1) and prove that this scheme maintain the positivity and the boundedness of the solutions of the studied system then we give remark about the asymptotically stable of this scheme. Section 4 is devoted to approximate the solutions of the proposed system using JGLSCM. In Section 5, numerical simulations for the studied model subjected to three different scenarios are reported to show the efficiency and the applicability of NSFDM and to show the accuracy of JGLSCM. Finally, a conclusion is given in Section 6.

2. Notations and preliminaries

In this section, necessary preparations for subsequent discussions are given. We divided this section into three parts. In the first part, we recall some necessary definitions and mathematical preliminaries of the fractional derivatives. In the second part, we define the NSFDM. We collect some important properties of Jacobi polynomials in the last part.

2.1. Fractional calculus definitions

In the labels, many various definitions were introduced for the fractional derivatives (see e.g., [6,56,57]). Usually, the Riesz fractional derivative is used to define the space-fractional derivative. Also, Grünwald–Letnikov operator, Riemann–Liouville operator and Caputo operator used to define the time-fractional derivatives. Nowadays the Caputo fractional derivative is the most common fractional derivative among applied scientists and engineers because it has the advantage of dealing with any initial value problem in a appropriate manner.

Definition 2.1. Let $\alpha \in \mathbb{R}^+$, the Caputo fractional derivative of order α are defined by (Caputo, 1967)

$$({}_0^cD_t^\alpha f)(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{f^{(n)}(x)}{(t-x)^{1-n+\alpha}}dx, \quad t>0, \tag{2}$$

where $f(x) \in C^n[0, \infty[$, $n = [\alpha] + 1$.

From the definition we can see that the derivative of a constant function using Caputo operator is zero, and

$${}_0^cD_t^\alpha t^\beta = \begin{cases} 0, & \text{for} \ \ \beta < \alpha, \\ \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}t^{\beta-\alpha}, & \text{for} \ \ \beta \geq \alpha. \end{cases} \tag{3}$$

Recall that when $\alpha \in \mathbb{N}$ the Caputo differential operator coincides with the usual differential operator of an integer order. Also, similar to the integer-order differentiation, Caputo's fractional differentiation is a linear operation; i.e.

$${}_0^cD_t^\alpha(\lambda f(t) + \gamma g(t)) = \lambda\ {}_0^cD_t^\alpha f(t) + \gamma\ {}_0^cD_t^\alpha g(t).$$

2.2. The nonstandard finite difference method

The technique of the NSFDM was firstly proposed by Mickens [31–34]. It is a method to construct a numerical discrete scheme for ordinary or partial differential equations (PDEs). The NSFDM is able to maintain the properties of the analytic solution of the original ODEs or PDEs with the following rules [33]: