Chaos, Solitons and Fractals 104 (2017) 806–812

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Probabilistic response analysis of nonlinear vibration energy harvesting system driven by Gaussian colored noise

Di Liu [a,*], Yong Xu [b,d,e], Junlin Li [c]

[a] *School of Mathematics, Shanxi University, Taiyuan, 030006, PR China*
[b] *Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an 710072, PR China*
[c] *School of Applied Science, Taiyuan University of Science and Technology, Taiyuan, 030024, PR China*
[d] *School of Mathematics and Information Science, Beifang University of Nationalities, Yinchuan 750021, PR China*
[e] *Potsdam Institute for Climate Impact Research, Potsdam 14412, Germany*

A R T I C L E I N F O

Article history:
Received 20 February 2017
Accepted 18 September 2017

Keywords:
Nonlinear vibration energy harvesting
Quasi-conservative averaging method
Mean-square electric voltage
Gaussian colored noise
Correlation time

A B S T R A C T

A new quasi-conservative stochastic averaging method is proposed to analyze the Probabilistic response of nonlinear vibration energy harvesting (VEH) system driven by exponentially correlated Gaussian colored noise. By introducing a method combining a transformation and the residual phase, the nonlinear vibration electromechanical coupling system is equivalent to a single degree of freedom system, which contains the energy-dependent frequency functions. Then the corresponding drift and diffusion coefficients of the averaged Itô stochastic differential equation for the equivalent nonlinear system are derived, which are dependent on the correlation time of Gaussian colored noise. The probability density function (PDF) of stationary responses is derived through solving the associated Fokker–Plank–Kolmogorov (FPK) equation. Finally, the mean-square electric voltage and mean output power are analytically obtained through the relation between the electric voltage and the vibration displacement, and the output power has a linear square relationship with the electric voltage, respectively. The main results on probabilistic response of VEH system are obtained to illustrate the proposed stochastic averaging method, and Monte Carlo (MC) simulation method is also conducted to show that the proposed method is quite effective.

1. Introduction

Vibration energy harvesting (VEH) is the process which energy is derived from external sources, captured and stored for small, wireless autonomous devices, like those used in wearable electronics and wireless sensor networks [1]. The basic operation principles of those devices are what convert available mechanical energy into electrical energy through the electromagnetic, piezoelectric, and electrostatic transduction mechanisms. Among them, the piezoelectric energy harvester plays a great role with its advantages in inducing excellent power densities, relatively high voltages, and low currents, and has already been widely used in engineering field.

Since the energy source for Vibration energy harvesters is present as ambient background and is free, and therefore the research of the dynamical behavior of VEH system has attracted more and more attention over the past 10 years. At the start of the study, most piezoelectric energy harvesters adopted the linear resonant model[2–4]. However, this model has a very narrow frequency bandwidth, which will result in the system become incapable of harvesting energy efficiently when the excitation frequency over a relatively wide range of the harvesters fundamental frequency. To remedy this problem, the unique advantages of nonlinearity of restoring force of the system had been suggested and applied. Comparing to the linear model, nonlinear model has a wider steady-state frequency bandwidth [5–7]. Among them, the mono-stable, bi-stable and tri-stable nonlinear oscillator, as the simplest nonlinearity model, had been used in the design of many VEH devices, with the addition of electromechanical coupling for the harvesting circuit. For instance, Triplett and Quinn [8] used the Poincare–Lindstedt perturbation method to analyze the response of a nonlinear lumped-model by including a nonlinear term in the electromechanical coefficient, and found that the nonlinearities in the electromechanical coupling can increase the harvested electrical power. Mann and Owens [9] used the magnetic interactions to create an electromagnetic VEH system of nonlinear bi-stable potential well and to validate the potential well escape phenomenon, and also testified the phenomenon will broaden the

* Corresponding author.
E-mail addresses: liudi@sxu.edu.cn (D. Liu), hsux3@nwpu.edu.cn (Y. Xu).

https://doi.org/10.1016/j.chaos.2017.09.027

frequency response through the theory and experiments. Stanton et al. [10] and Zhou et al. [11] investigated the dynamic responses of the bi-stable and tri-stable energy harvester by means of the harmonic balance method, respectively.

There is no doubt that the randomness exists in most real-world circumstances widely and therefore the nonlinear VEH, especially in stationary and non-stationary stochastic vibratory environments have attracted growing interest among researchers [12–16]. Specially, to better understand the effect of stochastic excitation on energy harvesting efficiency, some effective techniques had been proposed to investigate the stochastic response of VEH system with stochastic base acceleration. Jiang and Chen [17] used the Van Kampen expansion method to discuss the effects of the system parameters on the stochastic response of the piezoelectric coupling system, and recently they also presented the stochastic averaging method to analyze the response of nonlinear VEH system subject to external Gaussian white noise[18,19]. He and Daqaq [20,21] illustrated the effects of the potential energy function on the mean output power through the statistical linearization techniques and the steady-state approximation. Xu et al. [22] proposed a stochastic method combined with generalized harmonic transformation to analytically evaluate the mean-square electric voltage and mean output power for the nonlinear VEH system driven by Gaussian white noise. Recently, Fokou et al. [23] investigated the response, the stability and the reliability of a sandwiched piezoelectric buckled beam with axial compressive force under Gaussian white noise by means of stochastic averaging method.

Most studies of VEH systems have considered either sinusoidal excitations or Gaussian white noise excitations [24–28]. However, actual environmental excitations in applications can deviate from these idealizations. For this reason, the Ornstein–Uhlenbeck process, i.e., external colored noise had been attracting a lot of attention [29,30]. For example, Daqaq [31] employed the decouple approximate FPK equation methods to obtain the mean output power of a Duffing oscillator driven by exponentially correlated Gaussian noise. Mendez et al. [32] discussed the performance of a linear electromechanical energy harvesting system subjected to the arbitrary colored noise. Recently, a linear electromechanical oscillator with a random ambient excitation was considered as a VEH model, which had been researched by Bobryk and Yurchenko [33], they found that a parametric colored excitation can have a dramatic effect on the enhancement of the energy harvesting. However, there is a lock of effective analytical method to research the probabilistic response of nonlinear VEH system under the Gaussian colored noise excitation, and then to understand the effect of stochastic excitation on energy harvesting efficiency.

As mentioned above, our aims are to provide a stochastic approximate method to derive the probabilistic response and mean output power of nonlinear VEH system, and then to determine the effect of Gaussian colored noise on the energy harvesting efficiency. The paper is organized as follows. We first present the basic mathematical model of our consideration stochastic VEH system with Gaussian colored noise, and this stochastic model will be equivalent to a single degree of freedom system containing the energy-dependent frequency function through a method, which combines a transformation and the residual phase. In Section 3, the probabilistic response and the mean output power of nonlinear VEH system are derived under the assumption that the nonlinear VEH system is quasi-conservative. Results obtained from the proposed procedure are verified by direct numerical simulation results in Section 4. Furthermore, the effects of correlation time, the random excitations density, viscous damping coefficient and nonlinear stiffness coefficient on mean output power of the nonlinear VEH system is also investigated. A conclusion is given in the last section.

2. Nonlinear vibration energy harvesting system with Gaussian colored noise

The mathematical model in this investigation is considered to represent the dynamics of a class of piezoelectric vibratory energy harvester, which can be presented by a base-excited spring-mass-damper system coupled to a capacitive energy harvesting circuit, as shown in Fig. 1. The equation of motion can be described as [22,31]

$$m\ddot{\bar{X}} + c\dot{\bar{X}} + K_1\bar{X} + K_3\bar{X}^3 + \theta\bar{V} = -m\ddot{\bar{X}}_c(\tau), \tag{1a}$$

$$C_p\dot{\bar{V}} + \frac{1}{R}\bar{V} = \theta\dot{\bar{X}}, \tag{1b}$$

where $\bar{X}$ is the relative displacement of an inertial mass m. c, K_1 and K_3 denote the linear viscous damping coefficient, the linear, and nonlinear stiffness coefficients of VEH system, respectively. θ represents a linear electromechanical coupling coefficient. V is the voltage measured across an equivalent resistive load R, and C_p is the piezoelectric capacitance. $\ddot{\bar{X}}_c(\tau)$ is the stochastic base acceleration, and the dot represents the derivative with respect to time τ.

In this investigative, we consider a stochastic excitation source as Gaussian colored noise, which is a Gaussian noise with exponential correlated time. Applying the appropriate rescaling, one can obtain the following dimensionless model for a couple of nonlinear VEH system:

$$\ddot{X} + c\dot{X} + \omega_0^2 X + \gamma X^3 + \beta V = \xi(t), \tag{2a}$$

$$\dot{V} + \lambda V = \dot{X}, \tag{2b}$$

where the dot represents differentiation with respect to t. The stochastic base acceleration excitation $\xi(t)$ is the Gaussian colored noise, which has the following statistical properties

$$E[\xi(t)] = 0, \qquad R(\tau) = E[\xi(t)\xi(t+\tau)] = \frac{D_1}{\tau_1}\exp\left(-\frac{|\tau|}{\tau_1}\right). \tag{3}$$

Integrating the electric Eq. (2b), the following approximation expression of electric voltage can be written as

$$V(t) \doteq \int_0^t \dot{X}(s)\exp[-\lambda(t-s)]ds = \int_0^t \dot{X}(t-\tau)\exp(-\lambda\tau)d\tau. \tag{4}$$

By introducing a transformation

$$\begin{aligned} \mathrm{sgn}X\sqrt{U(X)} &= \sqrt{H}\cos\theta, \\ \dot{X} &= -\sqrt{2H}\sin\theta, \end{aligned} \tag{5}$$

where $U(X)$ and $H(t)$ are the potential energy and the total energy for undamped free motion, which cannot be explicitly expressed in this stage. $\theta = \theta(t) = \int_0^t \omega(H)ds + \phi(t)$ is the total phase, $\phi(t)$ denotes the residual phase which is slowly varying, and $\omega(H)$ denote energy-dependent frequency function. Therefore, the following approximations will be obtained by $\phi(t)$ is slowly varying

$$\begin{aligned} \theta(t-\tau) &= \int_0^{t-\tau}\omega(H)d\tau + \phi(t-\tau) \approx \theta(t) - \omega(H)\tau, \\ \dot{X}(t-\tau) &= -\sqrt{2H}\sin\theta(t-\tau) \\ &= \dot{X}\cos(\omega(H)\tau) + \mathrm{sgn}X\sqrt{2U(X)}\sin(\omega(H)\tau). \end{aligned} \tag{6}$$

Substituting Eq. (6) into Eq. (4), the nonlinear relationship between electric voltage and state variables can be written as