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Synchronized nonlinear patterns in electrically coupled Hindmarsh–Rose neural networks with long-range diffusive interactions

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ABSTRACT

Two electrically coupled Hindmarsh–Rose neural networks are considered, each including power-law long-range dispersive interactions. The whole dynamics of the system is reduced to a set of two coupled complex Ginzburg–Landau equations. The linear stability analysis of the plane wave solutions brings about the existence of two dynamical regimes that predict direct and indirect synchronization of the two networks, under the activation of modulational instability. The conditions for the latter to develop are discussed and used to observe numerically the synchronized longtime dynamics of action potentials, under the effect of both long-range intra-coupling and electrical inter-coupling parameters. Mainly, the synchronization criterion depends on the plane wave amplitudes and for some of their values, perfect and partial inter-network synchronization phenomena are observed. It is also found that indirect synchronization between adjacent networks requires local synchronization among neurons of the same fiber. This is discussed based on some further formulation of the synchronization error, additionally to the time series of action potentials. Some spatiotemporal behaviors of the corresponding bursts of spikes are also discussed using coupling parameters.

1. Introduction

Neural networks are favorable to cooperative behaviors, necessary for the efficient processing and transmission of information across the nervous system [1,2]. In fact, numerous operations are executed simultaneously in the brain due to the mechanism of synchronization of neural activities through phase locking and auto-generated oscillations and wave patterns [3]. Especially, wave formation and synchronization have been addressed intensively by several authors in order to unmask the hidden underlying mechanisms, in the presence of many factors such as time-delays [4,5], coupling strengths [1,6–8] and noise [9]. Temporal synchronization activities are of great importance in neural binding and information processing in the brain. Specifically, several brain disorders such as schizophrenia and Alzheimer's disease are closely related to abnormal synchronization. Mindful of the various factors that may enhance synchronized states in coupled oscillating entities, some general aspects of phase synchronization among oscillators through time-shifted common input were addressed recently [10]. Equally, effects of partial time-delay on phase synchronization in Watts–Strogatz small-world neuronal networks was studied, including its impact on the mean-firing rate of neurons [11]. In the process, it is nowadays well-established that neurons in networks may be coupled either chemically or electrically. Chemical synapses are responsible for the creation of circuits within the central nervous system, which portrays the complexity of brain connectivity that goes beyond the classical nearest-neighbor coupling between adjacent neurons. The neurotransmitter that is in fact released by a synapse may diffuse away from its target, therefore activating other synapses of the networks. In this category, one finds, for example, neurons in the brain cortex. Contrary to chemical synapses, electrical synapses, or gap junctions, are not due to the opening of the ion channels by chemical transmitters, but rather rely on direct coupling between neurons. They abound in the nervous system and may be found in the retina, the reticular nucleus of the thalamus, the hippocampus and the neocortex [12]. Chemical synapses being the main mode of communication

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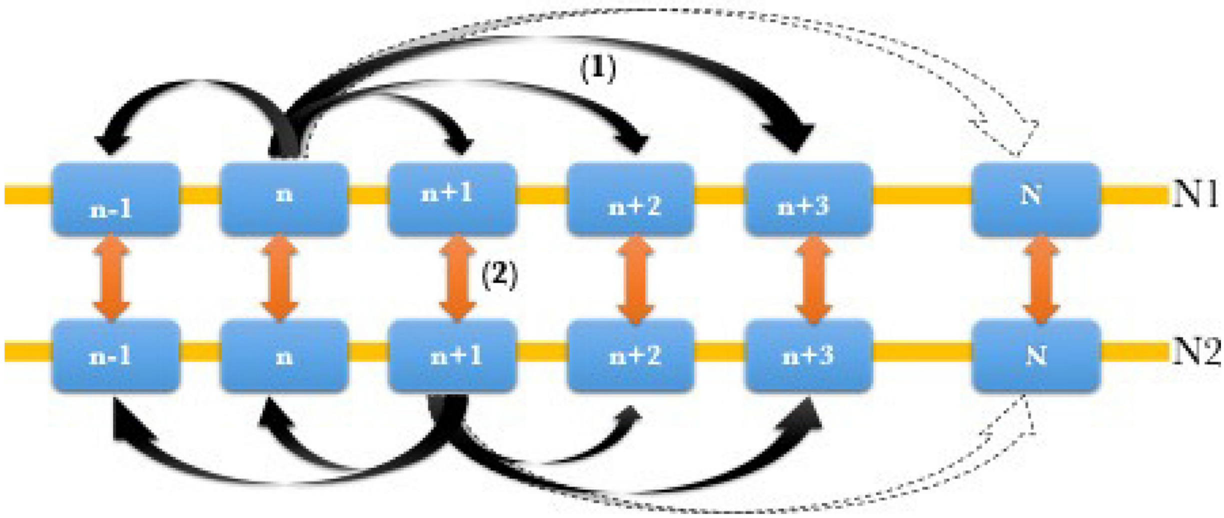

Fig. 1. Schematic diagram of the coupled networks **N1** and **N2. (1)** indicates the intra-coupling among neurons in the same network, while **(2)** picture the inter-network coupling between the two networks **N1** and **N2**. In individual networks, any neuron n may interact with the rest of the neurons in the lattice. In Eqs. (1), neuron **N1** corresponds to $\nu = 1$ and **N2** to $\nu = 2$.

between neurons, electrical synapses are favorable to synchronous behaviors in and between neural networks [13]. Otherwise, adjacent networks may directly communicate thanks to the direct electric connection between their elements. This mechanism might be useful in the process of recruiting damaged or dead neurons into conduction, and may even lead to some synchronous behaviors as recently reported by Maina et al. [14]. A simplified Hindmarsh–Rose (HR) model [15,16] was recently proposed, where global coupling was introduced in the form of power-law long-range (LR) interactions [17]. It was shown that under strong LR coupling, it was possible to observe disordered spatiotemporal patterns, therefore confirming the idea that strong interaction among bursting neurons may affect their synchronizability. In this purpose, the theory of modulational instability (MI) was applied, and regions of parameters for such behaviors to be observed was determined. Here, motivated by some experimental results suggesting that the number of interconnected neurons may be too large in the brain and lead to some inter-networks couplings [18–20], with inherent complex oscillating and bursting behaviors, we intend to explore the cooperative effects between electrically inter-coupled HR networks with power-law LR intra-coupling among neurons belonging to the individual networks (see Fig. 1). We show in fact that the competitive effects between the two kinds of coupling may bring about direct and indirect synchronous behaviors between the two networks.

MI is one of the direct mechanisms leading to the formation of wave patterns under the permanent competition between nonlinear and dispersive effects [21,22]. It has been addressed in a number of systems related to charge and energy transport in DNA and proteins [23–26], blood wave propagation in large vessels [27], calcium propagation in coupled cells [28,29] and nerve influx in neural networks [14,17,30], just to name a few. In the case of neural networks, it was also found that some events of synchronization were possible, but this could not be predicted analytically, via the linear stability analysis of MI. However, it was shown that solitons and nonlinear waves were suitable describing nerve impulses, with real application to one- and two-dimensional diffusive coupling [30]. Here, we establish a relationship between the formation of self-modulated waves and their synchronization using MI, and we show a strong correlation between analytical and numerical results, both for direct and indirect synchronization. It should be noticed that the problem of self-modulation in nonlinear science is straightforwardly related to phase and amplitude modulation. In neural network, however, phase synchronization is more appreciated than amplitude or complete synchronization. The subsequent solutions and collective behaviors may then be given in the form of asymptotic expansion, and an equation giving the modulation of the first-order amplitude may be derived. In our context, given the dissipative character of the studied system, the complex Ginzburg–Landau (CGL) equation is the simplest form of such amplitude equation as it is shown in this work. The rest of the paper is therefore articulated as follows: in Section 2, we introduce the model describing the dynamics of the two interacting HR networks, and we show, via the quasi-discrete approximation, that the dynamics of the whole system may be fully described by a set of two coupled complex Ginzburg–Landau (CCGL) equations. In Section 3, the theory of MI is used to find regions of parameter where trains of waves are expected, along with the condition for the dynamics of the two nerve fibers to be synchronized. In that direction, we find the condition for direct synchronization and we also propose the condition for indirect synchronization. In Section 4, the numerical verification of these predictions is performed, and one insists on the reliability of the analytical results. The synchronization between the two fibers is investigated via the calculation of errors, along with its response to intra- and inter-network interactions. In the case of indirect synchronization, a supplementary condition is used to detect synchronous states. In Section 5, some concluding remarks are given.

2. The coupled HR model and mathematical expansion

2.1. Model

HR neural networks are mainly described by a system of three nonlinear ordinary differential equations for the dimensionless dynamical variables $x(t)$, $y(t)$, and $z(t)$, where $x(t)$ represents the membrane potential, $y(t)$ and $z(t)$ refer to the rate of ion transport through fast and slow ion channels respectively [15,16]. Here we explore a modified HR model in which two discrete networks $\left(x_n^{(1)}(t), y_n^{(1)}(t), z_n^{(1)}(t)\right)$ and $\left(x_n^{(2)}(t), y_n^{(2)}(t), z_n^{(2)}(t)\right)$, with power-law intra-LR diffusive interactions [17,31], coupled via electrical synapses [32], as schematized in Fig. 1. The corresponding system