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Structure of growing complex networks coupling with the friendship and contact relations

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ABSTRACT

Understanding the structure and evolution of the social networks is a significant task. In this paper, we proposed a new evolving social networks model coupling the friendship networks and contact networks simultaneously. The mechanisms of growth and preferential attachment, random contact and disconnect, friendship decay and connection, acquaint by accident are involved. And the probabilities for a newly arrived node establishing connections to the existing vertices in each of the layers of the coupled social networks is set as a function of degrees of which at all layers. We establish and analyze the men-field model based on the evolving processes. Then the degree distributions on each of the layers are explored utilizing the mean-field model. Furthermore, we analyze and compare the degree distributions, the assortative or disassortative properties and the clustering coefficients between the two layers of the generate network by random simulations.

1. Introduction

A large of systems can be represented by complex networks, in which the vertices stand for the elements of the system and the edges note there is a relationship among them. Such as, in the Internet, whose nodes are the computers and whose edges represent the physical or wireless links between them. Equally in the social networks, where vertices are individuals and the edges are the social interactions between them. These complex networks in real worlds have a common structure even though the interactions between them are different. This common property is based on a universal self-organized mechanism: network growth and preferential attachment of connections, which can lead to a power-law degree distribution [1–3].

The topology structure of such evolving complex networks have been extensively investigated over the past decade, examples include the WWW [4], Internet [5], co-authorship networks [6], cooperation complex networks [7], human sexual-contact networks [8], social networks [9,10], various communication networks and so on, which show that the vertex connectivities of these complex networks follow a scale-free power-law distribution. In addition to the BA model proposed by Barabási and Albert [1], a lot of scale-free models have been proposed by considering more features of the networks [11–17]. Such as Holm et al. propose a network model which has both the perfect power-law degree distribution and the tunable high clustering [11]. The model with node and link removals also have been studied [13]. The effect of spatial topology on the promotion of cooperation dynamics are researched in the research works [14,15].

Furthermore, the study on evolving bipartite network model show the degree distribution of two different kinds of nodes both obey power-law form with adjustable exponents [16,17]. In Refs. [18,19], with both preferential and random attachments, the authors present an unweighted and weighted evolving network models, respectively. The system to describe crash as a consequence of some relatively simple local information-based individual behaviors is proposed and studied [20]. Many studies also focus attention on the community structure of the complex networks [21–24], and many research works establish the evolving network models with the neighborhood attachments [11,24], which think that the friends of a same individual are likely to know each other. Refs. [25,26] also give examples which show that the friendship network appear small world property, but these network dynamics ignore the growth property of the real word systems. And the topological structures have important effects on the epidemic spreading [27–29], Refs. [30–32] studied how the asymmetric contacts and adaptive behaviors affect the disease transmission on networks, respectively. As well as in [33,34], the interaction of the relationship between inducible defenses and herbivore outbreak

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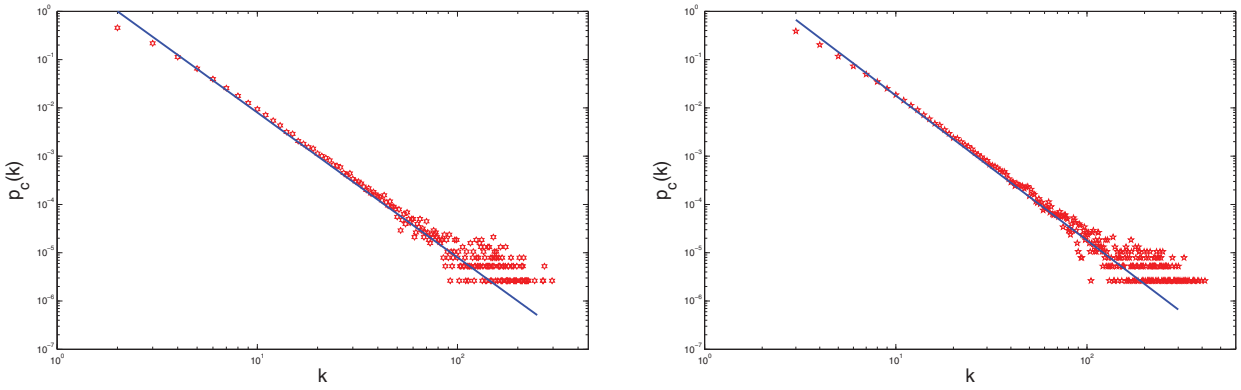

(a) The degree distribution of layer c. (b) The degree distribution of layer f.

Fig. 1. Plot of degree distributions of layer c and f while only consider the mechanisms of growth and preferential attachment with $m_0 = 5$. (a) The degree distribution of layer c with $m_c = 2$; (b) The degree distribution of layer f with $m_f = 3$. The blue lines denote the analytical results from Section 2, while red dotted lines are stochastic simulation results. The weight coefficients are by $c_1 = 0.2$ and $f_1 = 0.4$. The solid line and the dotted line represent the analytical and simulation results, respectively. Both of the degree distributions at layer c and f are power-law distribution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and the dynamical behaviors of an activator-inhibitor model with different sources are investigated. Moreover, epidemic dynamics with or without infectious force in latent period on random networks are studied in [35], and the summarize research works of the effect of vaccination and epidemics on networked populations are introduced in [36,37].

Though the complex network researchers have proposed a variety of evolving network models including kinds of social network evolutionary mechanism, they just investigated the social network models by looking at one type of relationship at a time, such as regarding the friendship network and contact network as separate networks. However, in real world, different types of edges can occur in the same social network. For example, while connections among individuals in friendship networks stand for they are acquaint with each other, the contact relationships among individuals may also exist. On the one hand, these two relationships should been treated as different, because friends may not always contact, a couple of individuals with contacts are also not necessarily friends. On the other hand, the topological properties of these two networks and their evolutions can depend on each other. Hence the real social network should contain both friendship and contact networks, that is to say, the same set of individuals in the social networks can be connected through friendship, contact relationships, and these relationships are changing all the time due to the effected of each other, so considering these links as being equivalent results into losing a lot of important information and obtaining inaccurate results, especially in the epidemic spreading area. Consequently, the social networks can be regarded as a special type of *multiplexes*, which is defined in [38]. Therefore the social networks composed of a same set of $N(t)$ vertices which can be connected by 2 different means (friendship and contact) of edges. And these formed 2 sub-networks are viewed as the different layers of the coupled networks, which can be called friend layer(f-layer) and contact layer(c-layer), respectively.

In this paper, we focus on the friendship and contact relationships on the social networks, our goal is to model a kind of growth social networks incorporating such two kinds of relationships among its basic individuals, which are the most important two factors to study the epidemic and information spreading on the social networks. Inspired the preferential attachment proposed by the classical BA model, in order to model the degrees at one layer can influence the degrees at another layer, in this paper the probabilities for a newly arrived node connecting to the existing vertices in each of the layers of the coupled social networks is a function of degrees of other nodes at all layers. While the growth mechanism of the social network is defined, the evolution of new acquaintances and contacts forming between individuals and old ones decaying in the social network are also depicted. In addition, the interplay between forming new friends and contacts is also enrolled to our model. We also aim to the model is simple enough that it allow us to derive the approximate analytical expressions of the basic characteristics of the network, such as the degree distributions, etc. Then we can use the mean field theory, which has been introduced to study fundamental characteristics of growing network models [1,3], to establish the rate equations that are special to our model. We solve them and obtain analytical expressions of both the degree distributions at each layer.

The outline of this paper is as follows. In Section 2, we introduce the evolution mechanisms of the network and give the generate network model involving the contact and friendship layers. In Section 3, the mean-field model based on the generate network steps is established. What is more, the analysis of the mean-field model at the trivial case only consider the growth and preferential mechanisms are given. In Section 4, we list the numerical simulations to study the topology structure of the generate network, including the degree distributions, clustering coefficient and mean nearest-neighbor degree at each layer. Finally, a brief conclusion is given in Section 5.

2. The model

We consider first a social network composed of $N(t)$ vertices which are connected to each other by means of edges belonging to 2 different type: friendship and contact relationship. Each class of edges can be represent as a separate layer. We can denote all the friendship and contact relationship links by f-layer and c-layer, respectively. Let V be the same set of vertices in the 2 layers of the social networks, and let E^f (or E^c) be the set of all the edges of type f (or c). Hence the social network can be represented as $\mathcal{G} = (V, E^f, E^c)$, and the size of the vertices is $|V| = N(t)$, the number of edges at layer f (or c) is $|E^f|$ (or $|E^c|$). Each of the 2 layers of the social network is an sub-network, we can give the definition of corresponding adjacency matrix $A^\alpha = \{a_{ij}^\alpha\}$ in layer α, where $a_{ij}^\alpha = 1$ if vertex i and j are connected through a link on layer α, where α is c or f, meaning the contact or friendship layer. Therefore the social network in this paper is fully specified by the vector of the