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# Nonlinear electromechanical energy harvesters with fractional inductance

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In this paper, an electromechanical energy harvesting system exhibiting the fractional properties and subjected to the harmonic excitation is investigated. The main objective of this paper is to discuss the system performance with parametric coupling and fractional derivative. The dynamic of the system is presented, plotting bifurcation diagram, poincaré map, power spectral density and phase portrait. These results are confirmed by using $0-1$ test. The harmonic balance method is used with the goal to provide the analytical response of the electromechanical system. The numerical simulation validates the results obtained by this analytical technique. In addition, replacing the harmonic by the random excitation, the impact of noise intensity, the fractional order derivatives $\kappa$ and the amplitude of the parametric coupling $\gamma$ is investigated in detail. It points out from these results that for the best choice of $D$, $\kappa$ and $\gamma$, the output power can be improved.



## 1. Introduction

In the past two decades, fractional calculus has attracted the attention of scientists and engineering, resulting to the development of many applications [1–3]. However, this field of research did not grow until recently, largely because the underlying mathematics was difficult. Thanks to the many methods for approximation of the fractional derivative and integral available in the literature nowadays, this barrier is considerably eliminated. Thus, fractional-order systems have been intensively studied in various areas namely, in biology, physics, chemistry, traffic systems, genetic algorithms and control systems [4–13]. Indeed, the concept of fractional derivative goes back to discussing that Leibniz and l'Hospital had over three under years ago about the half order derivatives. The interest accorded to this term is due to the experiments investigations which had shown, fractional order derivative appears to render real phenomena meaningful. For instance, in mechanical engineering, one used it to model viscoelastic properties in the physical system.

In physics, most particulary in the domain of energy harvesting, scientific research is mainly focused on enhancing the efficiency of the system. Many researchers groups [14–17] had considered nonlinear effects to reach widening the frequency bandwidth of the system. C.Nono et al. [19] used the Melnikov theory to discuss the performance of a bistable harvester by analyzing the critical condition for homoclinic bifurcation that could induce chaos in the system. Owens and Mann [20] discussed the effects of linear and nonlinear transduction and demonstrated that with a suitable design, nonlinear coupling is better than linear. Borowiec et al. [21] proposed a beam consisted of substrate and sandwiched with a tip mass which transduce the bending strains induced by the random horizontal displacement into electrical charge. They analyzed the efficiency of this nonlinear device by focusing on the region of stochastic resonance where beam motion has a large amplitude. Coccolo et al. [22] have studied the electrical response of a bistable system, by using a double-well Duffing oscillator, connected to a circuit through piezoceramic elements and driven by both a low and a high frequency forcing, where the high frequency forcing is the environmental vibration, while the low frequency is controlled by us. They showed that the response amplitude at the low-frequency increases, reaches a maximum and then decreases to a certain range of the high frequency forcing. They also demonstrated in their work that by enhancing the oscillations, we can harvest more electric energy.

Recently, a large amount of work in engineering vibrations showed that long-memory factor exists in many practical systems, which are difficult to be accurately described by integer-order models [23–28]. Bagley and Torvik [29–31] pointed out that half-order fractional derivative models can quite well describe the fre-

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quency dependence damping of viscoelastic materials. Kelly et al. [32] have applied the fractional Kelvin model to predict the seismic response of natural rubber bearings. Markris et al. [33] presented a fractional derivative Maxwell model for a viscous damper and validated their model using experimental results. Cao et al. [34] recently considered an energy harvesting system with fractional order viscoelastic material. They showed that the fractional order property of the material enhances high-energy chaotic motion as well as inter-well periodic oscillation. Kitio et al. [35] proposed an electromechanical energy harvesting system with a fractional order current voltage relation-ship for the electrical circuit and fractional power law in the restoring force of its mechanical part. They authors showed that under a single-well potential configuration, for a small amplitude of the perturbation, as the order of derivative increases, the resonant amplitude of mechanical vibration decreases while the bending degree remains fairly constant. For a large amplitude of the perturbation, the output power increased, this is due to the hardening effects. However, under a double-well configuration, the fractional power stiffness strongly affects the crossing well dynamics and consequently the output electrical power. Ducharne et al. [36] built and energy harvesting devices based on piezoelectric Ericsson cycles in a piezoceramic material. They showed that by coupling an electric field and mechanical excitation on Ericsson-based cycles, the amplitude of the harvested energy can be highly increased, and can reach a maximum close to 100 times its initial value. Several electromechanical models have been the subject of such study, in particular this of Oumbé et al. [37]. In this work, the authors studied the effect of a nonlinear inductance induced by the saturation of the magnetic circuit. Siewe et al. [38] worked on an unsaturated magnetic circuit where they focused on the study of dynamics of the model (study of chaos via the Melnikov method) and the energy transfer from the mechanical to electrical subsystem without interesting to the impact of inductance upon system performance. The present work is based on this model. An originality of this work comes from the fact that we have taken into account the fractional character of the inductance [35,39]. In this previous work, the authors assume that the inductance is linear and the magnetic field through the air-gap of the permanent magnet varies with the coil position. In this case, the voltage through the self is defined as $U = L\frac{di}{dt}$. Let us notice that in the experimental investigation, the coil exhibit the fractional properties [39]. Thus, the relationship between the current and voltage is defined as follows [35] $U_L = L\frac{\mathrm{d}^\kappa \mathrm{i}}{\mathrm{d}t^\kappa}$. The one of the purpose of this present paper is to investigate the impact of fractional inductance on the model propose in Ref. [38].

As pointed by Yamapi et al. [40,41], in certain circumstances, some parameters of the electromechanical device can vary with time because of the functioning constraints. This is particularly the case for the parameters of the electromagnetic coupling. In this present work, we consider that the magnetic field varies with time. This give rise to a parametric coupling which could play an important role in the improvement of the output power. The remain of the manuscript is organized as follows: Section 2 is devoted to the description of the system by a system equation. In Section 3, we evaluate analytical and numerically, the mechanical and electrical response of the system. This section is followed by the numerical simulation in Section 4. In Appendix, we have the conclusion.

## 2. The model and governing equations

As pointed by siewe et al. [38], the electromechanical device shown in Fig. 1 is composed of two fundamental parts: The mechanical part is composed of the mass $m$, the nonlinear spring and nonlinear damping, while the electrical subsystem is composed of fractional inductor $L$, a linear capacitor $C$ and the linear resistor $R$. We particularly consider the dissipative force with nonlinear dissipation term proportional to the power of velocity $(y')^3$. The expressions defining damping force is as follows: [43]

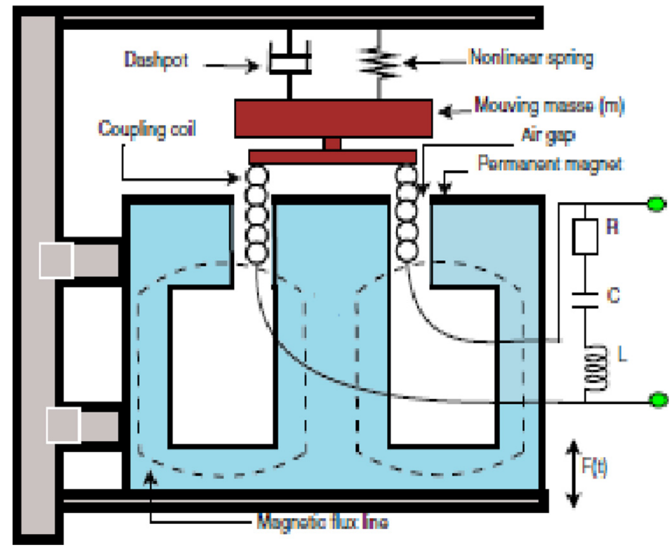


**Fig. 1.** Schematic model with the associated electric circuit.

$$f_d = c_1 y' + c_3 (y')^3. \tag{1}$$

Where $y'$ is the velocity of the mass, $c_1$ and $c_3$, the linear and nonlinear damping coefficients. The nonlinear damping introduced in this system is important insofar as it has been shown that it can improve efficiency in the context of EHS [42,43] Moreover, it is close to the reality because experimental studies have been done recently or it appears that nonlinear dissipation is the one, that offers better performances in terms of optimization. The mathematical expression of the magnetic field is defined as in Ref. [40]

$$B = B_0(1 + \gamma \cos(2\omega_1 t)), \tag{2}$$

where $B_0$ is the highest intensity that the field B reaches, $\gamma$ is the amplitude of the parametric coupling. The motion equation of the system is given as follows[38]:

$$\begin{array}{l} my'' + g(y, y') - lB_0\,(1 + \gamma \cos\,(2\,\omega_1\,t))q' = F(t) \\ LD_t^{\kappa+1} q + Rq' + \frac{q}{C} + lB_0\,(1 + \gamma \cos\,(2\,\omega_1\,t))y' = 0 \end{array} \tag{3}$$

with

$$g(y, y') = c_1 y' + c_3 (y')^3 + k_0 y + k_1 y^3.$$

where $(') = \frac{d}{dt}$, $y$ and $q$ are the displacement of the mass and charge respectively, $k_0$ and $k_1$ is the linear and nonlinear stiffness of the spring, while $l$ is the length of the air gap. Using the following transformation of coordinates:

$\omega_0^2 = \frac{k_0}{m}, y = lx, q = Q_0 z, \alpha = \kappa + 1$ and by letting the time variable $t = \frac{\tau}{\omega_0}$, the dimensionless equation is given by:

$$\begin{array}{l} \ddot{x} + f(x, \dot{x}) - \vartheta_m (1 + \gamma \cos(2\omega\tau))\dot{z} = F(\tau), \\ \dot{z} + \beta D_\tau^\alpha z + \mu_e z + \vartheta_e (1 + \gamma \cos(2\omega\tau))\dot{x} = 0 \end{array} \tag{4}$$

with

$$f(x, \dot{x}) = \mu_1 \dot{x} + \mu_3 \dot{x}^3 + \varrho x + \lambda x^3$$

and

$$\omega = \frac{\omega_1}{\omega_0}, \mu_1 = \frac{c_1 \omega_0}{k_0}, \mu_3 = \frac{l^2 c_3 \omega_0^3}{k_1}, \vartheta_e = \frac{l^2 B_0}{Q_0 R}$$

$$\lambda = \frac{l^2 k_1}{\omega_e{}^2 m}, \vartheta_m = \frac{B_0 \omega_0^3 Q_0}{k_0}, \mu_e = \frac{1}{\omega_0 RC}, \beta = \frac{\omega_0^\kappa L}{R}.$$