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Improved delay-dependent robust passivity criteria for uncertain neural networks with discrete and distributed delays

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ABSTRACT

This paper studies the problem of delay-dependent passivity for uncertain neural networks (UNNs) with discrete and distributed delays. Without considering free weighting matrices and multiple integral terms, which may cause more numbers of linear matrix inequalities (LMIs) and scalar decision variables. By constructing a suitable Lyapunov–Krasovskii functional (LKF) and combining with the reciprocally convex approach, some sufficient conditions are established in terms of LMIs. Compared with existing results, the derived criteria are more effective due to the application of delay partitioning approach which takes a full consideration of all available information in various delay intervals. Two simulation examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

During the past decades, neural networks (NNs) have been widely applied to static image processing, pattern recognition, fixed-point computation, associative memory, combinatorial optimization, etc [4–10]. Meanwhile, due to the finite speed of information processing and inherent communication time of neurons, time delay occurs commonly in many NNs, it may lead to oscillation, divergence, and even instability of NNs. Thus the stability of delayed NNs become an important issue. So far, the stability criteria of delayed NNs can be classified into two categories, that is, delay-independent ones [1–3] and delay-dependent ones [4–40]. The delay-independent stability conditions are usually more conservative than delay-dependent conditions since they include less information concerning the time delay, especially when the time delay is sufficiently small. For the delay-dependent case, we can choose an appropriate LKF for obtaining less conservative stability results. Hence, various improved schemes have been proposed to reduce conservatism in recent years. These include free-weighting matrix [11,12], augmented LKF [13] and so on. However, it is still hard to reduce the conservatism by further employing the identical LKFs. Recently, the delay-partitioning method has been proposed to investigate the stability of delayed NNs, which significantly reduce the conservatism of derived stability criteria in [9,12,14,23,33]. For example, in [9], by dividing the delay interval [0,d] into some subintervals with the same size, an improved stability condition for delayed NNs was obtained. Further, the authors in [14] investigated the delay-dependent stability issue for delayed NNs by using independent upper bounds of the delay derivative in each subinterval. In addition, in [33], the stability problem for a class of NNs with multiple time-varying delays was addressed by delay-partitioning and reciprocally convex approaches. However, it is worth noting that these methods have common shortcomings. For one thing, the relationship between time-varying delay and each subinterval is completely neglected. On the other hand, some useful information about neuron activation functions are also not sufficiently considered.

Moreover, it is well known that the dissipative theory plays an important role in the stability analysis of dynamical system, nonlinear control and other areas. Particularly, passivity as a special case of dissipative, which frequently used in control systems to prove the stability of the system. The passivity theory is intimately related to the circuit analysis and has received much attention from the control areas since 1970s. It has also been extensively applied in many physical systems such as signal processing, fuzzy control, sliding mode control [15] and networked control [16]. On the other hand, due to modelling error, external perturbation and

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parameter fluctuation, the NNs certainly involve some uncertainties such as perturbations and component variations, which will degrade the quality of system. Hence, in [27–30,33,36], the authors considered the problem of delay-dependent passivity for uncertain continuous-time NNs with both discrete and distributed delays. Further, a new integral inequality based on Wirtinger's inequality was proposed in [20], which gives a tighter upper bound. Therefore, it is necessary and meaningful to study the issue of delay-dependent passivity for UNNs with both discrete and distributed delays.

Motivated by the above considerations, in this paper, the problem of delay-dependent passivity for UNNs with both discrete and distributed delays is studied. Without considering free weighting matrices and multiple integral terms, which may cause more numbers of LMIs and scalar decision variables. By constructing a suitable Lyapunov–Krasovskii functional and combining with the reciprocally convex approach, some sufficient conditions are established in terms of LMIs. Compared with existing results, the derived criteria are more effective due to the application of delay partitioning approach which takes a full consideration of the all available information in various delay intervals. Finally, two simulation examples are given to illustrate the effectiveness of the proposed method.

Notation: Throughout this paper, the superscript T denotes the transposition and the notation $X \geq Y$ ($X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (positive definite). R^n and $R^{n\times n}$ denote n-dimensional Euclidean spaces and the set of all $n \times n$ real matrices, respectively. I is the identity matrix. The notation $*$ always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

2. Problem statement and preliminaries

Consider the following uncertain neural network with discrete and distributed delays:

$$\begin{cases} \dot{x}(t) = -(A+\Delta A(t))x(t) + (W+\Delta W(t))g(x(t)) \\ \qquad + (W_1+\Delta W_1(t))g(x(t-\tau(t))) \\ \qquad + (W_2+\Delta W_2(t))\int_{t-\tau(t)}^{t} g(x(s))ds + u(t) \\ y(t) = g(x(t)) \\ x(t) = \phi(t), t\in[-d,0]. \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ is the neuron state vector, $u(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^T \in R^n$ is the input vector, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t))]^T$ denotes the neuron activation function, $A = diag(a_1, a_2, \dots, a_n) > 0$ is a positive diagonal matrix, $W = (b_{ij})_{n\times n}$, $W_1 = (c_{ij})_{n\times n}$ and $W_2 = (d_{ij})_{n\times n}$ are the interconnection matrices representing the weighting coefficients of neurons and $y(t)$ is the output vectors. The continuous function $\phi(t)$ is the initial condition. $\Delta A(t)$, $\Delta W(t)$, $\Delta W_1(t)$ and $\Delta W_2(t)$ are unknown matrices that represent the time-varying parameter uncertainties and the delay $\tau(t)$ is a time-varying function with $0 \leq \tau(t) \leq d$, $\dot{\tau}(t) \leq \mu$. $\Delta A(t)$, $\Delta W(t)$, $\Delta W_1(t)$ and $\Delta W_2(t)$ are assumed to be of the form:

$$\begin{aligned} &[\Delta A(t) \quad \Delta W(t) \quad \Delta W_1(t) \quad \Delta W_2(t)] \\ &= [G_1F_1(t)E_1 \quad G_2F_2(t)E_2 \quad G_3F_3(t)E_3 \quad G_4F_4(t)E_4] \end{aligned} \tag{2}$$

where G_i and E_i $(i = 1, 2, 3, 4)$ are known real constant matrices and $F_i(t)$ $(i = 1, 2, 3, 4)$ are unknown time-varying matrix functions satisfying

$$F^T(t)F(t) \leq I, \quad \forall t \geq 0. \tag{3}$$

It is assumed that all elements $F_i(t)$ are Lebesque measurable. The uncertain matrices $\Delta A(t)$, $\Delta W(t)$, $\Delta W_1(t)$ and $\Delta W_2(t)$ are said to be admissible if (2) and (3) hold.

For convenience, system (1) can be rewritten as

$$\begin{cases} \dot{x}(t) = -A(t)x(t) + W(t)g(x(t)) + W_1(t)g(x(t-\tau(t))) \\ \qquad + \int_{t-\tau(t)}^{t} g(x(s))ds + u(t) \\ y(t) = g(x(t)) \\ x(t) = \phi(t), t\in[-d,0]. \end{cases} \tag{4}$$

where $A(t) = A + \Delta A(t)$, $W(t) = W + \Delta W(t)$, $W_1(t) = W_1 + \Delta W_1(t)$, $W_2(t) = W_2 + \Delta W_2(t)$.

Throughout this paper, we suppose that the activation function satisfies the following assumption.

Assumption 2.1. [17] The neuron activation function $g_i(\cdot)$ in (1) is continuous and satisfies

$$k_i^- \leq \frac{g_i(\alpha_1) - g_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq k_i^+, \alpha_1, \alpha_2 \in R, \alpha_1 \neq \alpha_2, \quad for \quad i = 1, 2, \cdots, n \tag{5}$$

Where $k_i^-, k_i^+ (i = 1, 2, \cdots, n)$ are known real scalars, and $g_i(0) = 0, i = 1, 2, \cdots, n$. From (5), if $\alpha_2 = 0$, then we have $k_i^- \leq \frac{g_i(x_i(t))}{x_i(t)} \leq k_i^+, \alpha \neq 0$. Thus, under this assumption, the following inequality holds for any diagonal matrix $H > 0$:

$$x^T(t)KHKx(t) - g^T(x(t))Hg(x(t)) \geq 0,$$

Where $K = diag(k_1, k_2, \dots, k_n)$, $k_i = max(|k_i^-|, |k_i|^+)$.

To end this section, we introduce the following definition and lemmas, which will play an important role in the proof of the main results.

Definition 2.1. [22] The neural network (4) is said to be passive if there exists a scalar $\gamma > 0$ such that for all $t_f \geq 0$

$$2\int_0^{t_f} y(s)^T u(s)ds \geq -\gamma \int_0^{t_f} u(s)^T u(s)ds \tag{6}$$

under the zero initial condition.

Lemma 2.1. [20]. *For any a given matrix $Q > 0$, the following inequality holds for continuously differentiable function $x(t)$ in $[a, b] \in R^n$:*

$$-(b-a)\int_a^b \dot{x}^T(s)Q\dot{x}(s)ds \leq -[x(b)-x(a)]^T Q[x(b)-x(a)] -3\xi(t)^T Q\xi(t)$$

where $\xi(t) = x(b) + x(a) - (2/(b-a))\int_a^b x(s)ds$.

Lemma 2.2. [26]. *For any vectors x_1, x_2, matrix S and symmetric matrix Q, and real scalar $0 \leq \alpha \leq 1$ satisfying that $\begin{bmatrix} Q & S \\ * & Q \end{bmatrix} \geq 0$, the following inequality holds:*

$$-\frac{1}{\alpha}x_1^T Q x_1 - \frac{1}{1-\alpha}x_2^T Q x_2 \leq -\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} Q & S \\ * & Q \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Lemma 2.3. [32]. *For the given matrices D, E and F with $F^TF \leq I$ and positive scalar $\varepsilon > 0$, the following inequality holds:*

$$DFE + (DFE)^T \leq \varepsilon DD^T + \varepsilon^{-1}E^TE.$$

3. Main results

In this section, by using delay partitioning approach, we will derive new delay-dependent passivity criteria for system (4) with mixed time-varying delays.