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Impacts of additional food on diffusion induced instabilities in a predator-prey system with mutually interfering predator



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ABSTRACT

In this paper, diffusion driven pattern forming instabilities in a predator-prey system with mutually interfering predators described by the Beddington-DeAngelis type functional response, are investigated in the presence of additional food for predators. Conditions for Hopf, Turing and wave instabilities are investigated around the coexisting equilibrium point analytically. Numerical simulation results are presented to show different types of spot, stripe and their mixture patterns. Different spatial domains in the parameter space are plotted. The existence and non-existence of positive, non-constant, steady states of the reaction-diffusion model are established. It is observed that spatio-temporal pattern of a predator prey system can change significantly depending upon the parameters related to additional food. We can conclude from our study, that the reasons of appearance of different spatio-temporal patterns in the real life ecological systems may be due to variation of additional food.

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1. Introduction

Spatio-temporal patterns appear almost everywhere in nature and their description and understanding still raise important and basic questions. The emergence of the spatial pattern is in general an outcome of the interplay between spatial structure and self-organization, although each of them can be a dominant factor in the dynamics and structure of a specific ecosystem. The study of pattern formation in reaction-diffusion (RD) systems is a very active research area since the seminal work of Turing [1]. Turing pointed out that to generate spatial patterns, a reaction diffusion system should contain at least two reactive species that diffuse at very different rates: one slowly diffusing substance and other rapidly diffusing substance.

After the pioneer work of Turing, understanding the mechanisms behind self-organized spatial pattern formation has been received great deal of attention in several branches of science like chemical reaction system [2–5], morphogen concentrations [6–8], neuronal networks [9,10], hydrodynamics [11,12] etc.

Segel and Jackson [13] applied first the Turing's ideas about pattern formation on population dynamics. After Segel and Jackson [13] pattern formation in variety of spatio-temporal predator-prey models were reported [14–26]. Spiral waves are ubiquitous features of nature and have been observed in the nervous system, in turtle

visual cortex [27], in rodent brain slices [28]. Spiral waves have been found to play an important role in cardiac arrhythmia [29]. Spiral waves are also found in population dynamics [30,31]. Study of ecological systems in different biological environments are very useful to understand the real world population dynamics. Recently, the study of Sun [32] to investigate the Allee effect induced pattern forming instabilities in population dynamics was a step towards it. Moreover, influence of isolation degree [33] and time delay [34,35] in ecological models with spatial diffusion were also investigated by Sun et al.

Additional food is an important component of most predators diet, although they receive less attention than prey in the scientific literature [36–39]. The availability of suitable additional food (non-prey food) in an ecosystem can have significant impact on the spatio-temporal dynamics of a predator-prey system [30]. The consequences of providing additional food to predator on the spatio-temporal dynamics of predator prey system may be very useful in biological control (such as species conservation and pest management) [38,40,41]. In recent years, many biologists, experimentalists, and theoreticians investigated the consequences of providing additional food to predators in predator-prey systems [42–44]. Since the presence of additional food can modify the prey predator interaction, therefore it can play a vital role in the pattern formation in an ecosystem [30]. These facts motivate us to study a simple reaction-diffusion model with two species in the presence of additional food for predators in this work.

Mutual interference denotes the adverse effects of predator density on the instantaneous success of individual predators. Mu-

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tual interference within species is common in nature and it has strong impacts on the community dynamics [40,45–47]. Beddington [48] was the first to call attention to the effects of mutual interference between predators on searching efficiency and modified the Holling type II functional response. The impacts of mutual predator interference in patchiness through an interplay between local population processes and diffusion were reported earlier [45,46]. The effects of diffusion on additional food provided predator-prey system with mutually interfering predators is an interesting area of investigation.

The objective of this paper is to determine the impacts of spatial diffusion in an additional food provided to predator of a predator-prey system with mutually interfering predators. Specifically, we have modified the Srinivashu et al. [40] model introducing, spatial diffusion in the model and investigate diffusion induced pattern forming instabilities in the model. Different types of patterns like spots, stripes, or their mixture occurs depending on the additional food parameters as well as the strength of the mutual inference among the predator.

The paper is organized as follows: In Section 2 we introduce a spatio-temporal food chain model with Beddington-DeAngelis type functional response where predator population is supported by some additional food. In Section 3 we investigate the existence and stability conditions of the stationary points in the absence of spatial diffusion as well as in the presence of diffusion and derived the conditions for Hopf bifurcation and Turing bifurcation. In the next Section 4 we have reported the numerical simulation results in a square domain with zero flux boundary conditions. Finally, in Section 5 we draw the conclusion.

2. Mathematical model

Recently dynamics of additional food provided predator-prey system with mutually interfering predators was reported by Prasad et al. [40]. They proposed a predator prey model taking Beddington-DeAngelis [48] type functional response and they have not reported the effected of spatial diffusion. The compact form of the model proposed by Prasad et al. [40] is following,

$$\begin{aligned} \frac{dX}{dt} &= rX \left(1 - \frac{X}{K}\right) - \frac{cXY}{a + \alpha A + X + \rho Y} \\ \frac{dY}{dt} &= \frac{b(X + \eta A)Y}{a + \alpha A + X + \rho Y} - mY \end{aligned} \quad (1)$$

where X, Y are the biomass of prey and predator respectively. The parameters r, K represent the intrinsic growth rate and carrying capacity of the prey respectively, c is the maximum rate of predation, a is the half-saturation value of the predator in the absence of additional food, α be the ratio between the handling times of additional food and the prey, A is the biomass of additional food, which is uniformly distributed in the habitat, ρ measures the strength of mutual interference among the predators, b maximum birth rate of the predator due to consumption of the food perceived by the predator, η is the ability of the predators to detect additional food relative to the prey and m is the mortality rate of predators in the absence of prey. If h_A and e_A respectively represent the handling time of the predator per unit quantity of the additional food and constant that signifies the predators movement rate while searching to detect the additional food, then α is equal to the ratio between the handling times towards the additional food and the prey i.e., $\alpha = \frac{h_A}{h_X}$. On the other hand $\eta = \frac{e_A}{e_X}$, can infer that it represents the effectual ability of the predator to detect additional food relative to the prey. Thus, the term ηA represents the quantity of additional food perceptible to the predator relative to prey. If A is taken to be zero, then the system (1) will reduce to well-known

Beddington-DeAngelis model [48]. Now re-defining

$$X_{new} = \frac{X_{old}}{a}, \quad t_{new} = r t_{old}, \quad Y_{new} = \frac{c Y_{old}}{ar}.$$

The system (1) can be transformed to the following non-dimensional form,

$$\begin{aligned} \frac{dX}{dt} &= X \left(1 - \frac{X}{\gamma}\right) - \frac{XY}{1 + \alpha\xi + X + \epsilon Y} \\ \frac{dY}{dt} &= \frac{\beta(X + \xi)Y}{1 + \alpha\xi + X + \epsilon Y} - \delta Y \end{aligned} \quad (2)$$

$$\text{where } \gamma = \frac{K}{a}, \quad \beta = \frac{bt}{r}, \quad \delta = \frac{m}{r}, \quad \xi = \frac{\eta A}{a}, \quad \epsilon = \frac{\rho}{c}.$$

In the presence of spatial self-diffusion, the model (2) takes the following form:

$$\begin{aligned} \frac{\partial X}{\partial t} &= X \left(1 - \frac{X}{\gamma}\right) - \frac{XY}{1 + \alpha\xi + X + \epsilon Y} + D_1 \nabla^2 X \\ \frac{\partial Y}{\partial t} &= \frac{\beta(X + \xi)Y}{1 + \alpha\xi + X + \epsilon Y} - \delta Y + D_2 \nabla^2 Y \end{aligned} \quad (3)$$

where $D_1 > 0$ and $D_2 > 0$ are the diffusion coefficients, $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$ is the Laplacian operator with x and y are the space variables. We shall investigate the effect of the spatial diffusion with the zero flux boundary condition.

3. Analytic results

3.1. Analysis of local dynamics

The equilibrium points of the non-diffusive system (2) are trivial equilibrium point $E_0 \equiv (0, 0)$, axial equilibrium points $E_1 \equiv (\gamma, 0)$, $E_2 \equiv \left(0, -\frac{\delta - \beta\xi + \alpha\xi\delta}{\delta\epsilon}\right)$ and coexisting equilibrium $E_3 \equiv (X_1^*, Y_1^*)$, $E_4 \equiv (X_2^*, Y_2^*)$ where

$$\begin{aligned} X_{1,2}^* &= \frac{\zeta_1 \mp \sqrt{\zeta_1^2 + 4\zeta_2\gamma\beta\epsilon}}{2\beta\epsilon} \\ Y_{1,2}^* &= \frac{(\beta - \delta)X_{1,2}^* - (\delta - \beta\xi + \alpha\xi\delta)}{\delta\epsilon} \\ \zeta_1 &= \beta\epsilon(\gamma - \xi) - \gamma(\beta - \delta) \\ \zeta_2 &= \delta - \beta\xi + \alpha\xi\delta + \beta\xi\epsilon. \end{aligned} \quad (4)$$

We observe that the interior point E_4 will exist when

$$\zeta_2 > 0 \text{ and } X_2^* > \frac{\delta - \beta\xi + \alpha\xi\delta}{\beta - \delta}. \quad (5)$$

In this paper, we focus only on the interior equilibrium point $E_4 \equiv (X_2^*, Y_2^*) \equiv E^* \equiv (X^*, Y^*)$ (say). It is easily observable that equilibrium points are the functions of additional food dependent parameters as well as the mutual interfering to the predator. Therefore, choosing suitable quality and quantity of additional food or mutual interfering parameter we can change the equilibrium point according to our preference. Now linearizing the system (2) about the equilibrium point E^* we obtain

$$\begin{aligned} A_0 &= \begin{pmatrix} \frac{X^*(\gamma - 1 - \alpha\xi - 2X^* - \epsilon Y^*)}{\gamma(1 + \alpha\xi + X^* + \epsilon Y^*)} & -\frac{X^*(1 + \alpha\xi + X^*)}{(1 + \alpha\xi + X^* + \epsilon Y^*)^2} \\ (\beta - \delta)\left(1 - \frac{X^*}{\gamma}\right) & -\delta\epsilon\left(1 - \frac{X^*}{\gamma}\right) \end{pmatrix} \\ &= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \text{ (say)}. \end{aligned}$$

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