Chaos, Solitons and Fractals 103 (2017) 246–255

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Periodic solutions of discrete time periodic time-varying coupled systems on networks

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ARTICLE INFO

Article history:
Received 9 September 2016
Revised 17 March 2017
Accepted 13 June 2017

Keywords:
Periodic solutions
Discrete time
Time-varying coupled systems
Networks

ABSTRACT

In this paper, we consider the existence of periodic solutions for discrete time periodic time-varying coupled systems on networks (DPTCSN). Some novel sufficient conditions are obtained to guarantee the existence of periodic solutions for DPTCSN, which have a close relation to the topology property of the corresponding network. Our approach is based on the continuation theorem of coincidence degree theory, generalized Kirchhoff's matrix tree theorem in graph theory, Lyapunov method and some new analysis techniques. The approach is applied to the existence of periodic solutions for discrete time Cohen–Grossberg Neural Networks (CGNN). Finally, an example and numerical simulations are provided to illustrate the effectiveness of our theoretical results.

1. Introduction

Complex networks are all around us in the real world nowdays. Discrete time coupled systems on networks as a mathematical framework could describe many complex networks in science and engineering, such as complex biological systems, neural networks, chemical systems, etc. [1–4]. On the other hand, periodic phenomena exist widely in biological systems such as the seasonal effects of weather, food supplies, etc., as well as electronic systems and neural networks. In this paper, we investigate the model of discrete time periodic time-varying coupled systems on networks (DPTCSN) as follows.

$$x_i(n+1) = f_i\big(x_i(n), n\big) + \sum_{j=1}^{l} b_{ij}(n) H_{ij}\big(x_j(n)\big), \quad i \in \mathbb{L}, \quad n \in \mathbb{N}, \qquad (1)$$

where $f_i : \mathbb{R}^{m_i} \times \mathbb{N} \to \mathbb{R}^{m_i}$ are continuous functions satisfying $f_i(\cdot, n) = f_i(\cdot, n+\omega)$ $(\omega \in \mathbb{Z}^+)$, $b_{ij}(n)$ represent the ω-periodic time-varying coupling strength, i.e. $b_{ij}(n) = b_{ij}(n+\omega)$ and $H_{ij}\big(x_j\big) : \mathbb{R}^{m_j} \to \mathbb{R}^{m_i}$ stand for normalized interference functions.

Exploring the global dynamics of DPTCSN is generally a challenging and difficult task as the following list of possible complications illustrates. Firstly, structural complexity: the global dynamics of DPTCSN do not only depend on every vertex system but also rely on the topology property of the networks structural. Secondly, dynamical complexity: every vertex could be described by nonlinear difference equations. It is well known that, compared with the continuous time systems, the discrete time ones are more difficult to deal with. Usually, some simple difference equations can always produce very complicated dynamics. Thirdly, connection diversity: the links between vertices could have different weights and the weight could be changing over time. Among the global dynamics behavior of DPTCSN, periodicity is the main one. In fact, the existence of a periodic solution as a similar character played by the equilibrium of the autonomous systems is a very basic and important issue in the study of DPTCSN. Motivated by the above discussions, investigated the existence of periodic solutions for DPTCSN is a splendid work and some new methods should be recommended.

Lots of approaches are used to investigate periodic solutions including many fixed point theorems, the upon and lower solution method, etc. Recently, with the help of a powerful technique which is the continuation theorem of coincidence degree theory, a lot of good results concerned with the existence of periodic solutions for discrete time systems and continuous time systems are obtained (see Refs. [5–10] and the references therein). However, how to acquire the priori estimate of unknown solutions to the equation $Lx = \lambda Nx$ is still a difficult issue and many scholars obtain the priori bounds by employing the inequality $|x(t)| \le |x(t_0)| + \int_0^{\omega} |\dot{x}(t)| \mathrm{d}t$ and matrix's spectral theory in the previous literatures. But, applying these previous approaches to acquire the priori estimate of unknown solutions to the equation $Lx = \lambda Nx$ for DPTCSN is very difficult due to its inherently difficulties discussed above. Fortunately, Li and Shuai in [11], use graph theory

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http://dx.doi.org/10.1016/j.chaos.2017.06.012
0960-0779/

method to investigate the global-stability problem for coupled systems of differential equations on networks. Based on the method in [11], many scholars have acquired lots of results about stability for different coupled systems, such as discrete time coupled systems [4,12], stochastic coupled systems [13,14], and delay coupled systems [15,16], etc. However, to the authors' knowledge, few people use this technique to investigate the existence of periodic solutions for DPTCSN. The main contribution and novelties of the current work are as follows.

1. Some new techniques based on Lyapunov method, generalized Kirchhoff's matrix tree theorem in graph theory and analysis skills for the priori estimate of unknown solutions to the equation $Lx = \lambda Nx$ are provided.
2. By employing the continuation theorem of coincidence degree theory, Lyapunov method and generalized Kirchhoff's matrix tree theorem in graph theory, some sufficient conditions are obtained which have a close relation to the topology property of the network's structure.

The tree of this paper is the following. In Section 2, some useful notations and basic preliminaries are given. In Section 3, some sufficient conditions for the existence of periodic solutions of DPTCSN are obtained. In Section 4, our approach is applied to discrete time Cohen–Grossberg Neutral Networks (CGNN). In Section 5, an example and numerical simulations are given to show the effectiveness and feasibility of our results. Finally, conclusions are presented in Section 6.

2. Preliminaries

In this section, we shall summarize some useful notations, basic concepts and lemmas in the following which will be used throughout this paper.

2.1. Notations

Throughout this paper, we denote ω a positive integer, and $I_\omega = \{0, 1, \cdots, \omega - 1\}$. Let $\Delta f(n) = f(n+1) - f(n)$, $\mathbb{L} = \{1, 2, \cdots, l\}$, $\mathbb{N} = \{0, 1, 2, \cdots\}$, $\mathbb{R}_+^1 = [0, +\infty)$, and $\mathbb{Z}^+ = \{1, 2, \cdots\}$. Write $\overline{f} = \max\limits_{n \in I_\omega} f(n)$, $\underline{f} = \min\limits_{n \in I_\omega} f(n)$ and $\hat{f} = \frac{1}{\omega} \sum_{n=0}^{\omega-1} f(n)$. Let $m = \sum_{i=1}^{l} m_i$, $m_i \in \mathbb{Z}^+$. Set $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote n–dimensional real space and $n \times m$-dimensional real matrix space, respectively. The transpose of vectors and matrices is denoted by superscript "T". For vector $y = (y_1, y_2, \cdots, y_n)^{\mathrm{T}} \in \mathbb{R}^n$, $|y|$ denotes the Euclidean norm $|y| = (\sum_{i=1}^{n} y_i^2)^{1/2}$. Denote by $C(\mathbb{R}^d \times \mathbb{N}; \mathbb{R}_+^1)$ the family of all real-valued nonnegative functions $V(x, n)$ denoted on $\mathbb{R}^d \times \mathbb{N}$ such that they are continuously in x and n. Other notations will be explained where they first appear.

2.2. Graph theory

We introduce some basic concepts on graph theory [17,18]. A directed graph or digraph $\mathcal{G} = (H, E)$ contains a set $H = \{1, 2, \cdots, l\}$ of vertices and a set E of arcs (i, j) leading from initial vertex i to terminal vertex j. A subgraph $\mathcal{H}$ of $\mathcal{G}$ is said to be spanning if $\mathcal{H}$ and $\mathcal{G}$ have the same vertex set. A digraph $\mathcal{G}$ is weighted if each arc (j, i) is assigned a positive weight $a_{ij}(n)$, for $n \in \mathbb{N}$. In our convention, $a_{ij}(n) > 0$ if and only if there exists an arc from vertex j to vertex i in $\mathcal{G}$. The weight $W(\mathcal{H})$ of a subgraph $\mathcal{H}$ is the product of the weights on all its arcs. A directed path $\mathcal{P}$ in $\mathcal{G}$ is a subgraph with distinct vertices $\{i_1, i_2, \cdots, i_m\}$ such that its set of arcs is $\{(i_k, i_{k+1}) : k = 1, 2, \cdots, m-1\}$. If $i_m = i_1$, we call $\mathcal{P}$ a directed cycle. A connected subgraph $\mathcal{T}$ is a tree if it contains no cycles, directed or undirected. A tree $\mathcal{T}$ is rooted at vertex i, called the root, if i is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph $\mathcal{Q}$ is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle. Given a weighted digraph $\mathcal{G}$ with l vertices, define the weight matrix $A(k) = (a_{ij}(n))_{l \times l}$ whose entry $a_{ij}(n)$ equals the weight of arc (j, i) if it exists, and 0 otherwise. Denote the directed graph with weight matrix $A(n)$ as $(\mathcal{G}, A(k))$. A digraph $\mathcal{G}$ is strongly connected if for any pair of distinct vertices, there exists a directed path from one to the other. A weighted digraph $(\mathcal{G}, A(n))$ is said to be balanced if $W(\mathcal{C}) = W(-\mathcal{C})$ for all directed cycles $\mathcal{C}$ and $n \in \mathbb{N}$. Here, $-\mathcal{C}$ denotes the reverse of $\mathcal{C}$ and is constructed by reversing the direction of all arcs in $\mathcal{C}$.

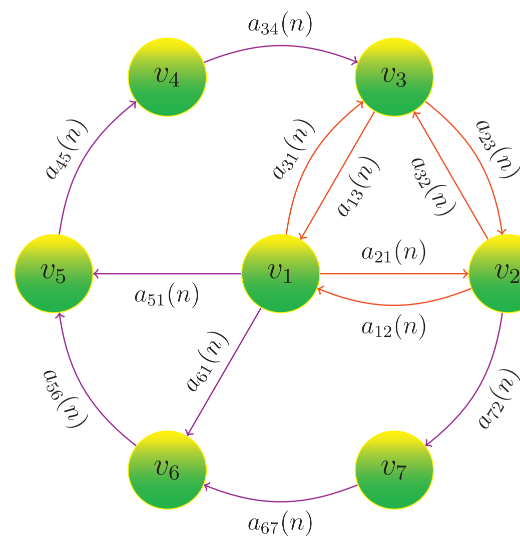

Fig. 1. A balanced digraph $(\mathcal{G}, A(n))$ with 7 vertices.

For example, consider a weighted digraph with 7 vertices (see Fig. 1), where $A(n)$ is a 7×7 matrix as

$$\begin{pmatrix} 0 & a_{12}(n) & a_{13}(n) & 0 & a_{15}(n) & a_{16}(n) & 0 \\ a_{21}(n) & 0 & a_{23}(n) & 0 & 0 & 0 & 0 \\ a_{31}(n) & a_{32}(n) & 0 & a_{34}(n) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{45}(n) & 0 & 0 \\ a_{51}(n) & 0 & 0 & 0 & 0 & a_{56}(n) & 0 \\ a_{61}(n) & 0 & 0 & 0 & a_{65}(n) & 0 & a_{67}(n) \\ 0 & a_{72}(n) & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad n \in \mathbb{N}.$$

This digraph is balanced if and only if $a_{31}(n)a_{23}(n)a_{12}(n) = a_{13}(n)a_{21}(n)a_{32}(n)$ hods for any $n \in \mathbb{N}$.

For a unicyclic graph $\mathcal{Q}$ with cycle $\mathcal{C}_\mathcal{Q}$, let $\tilde{\mathcal{Q}}$ be the unicyclic graph obtained by replacing $\mathcal{C}_\mathcal{Q}$ with $-\mathcal{C}_\mathcal{Q}$. Suppose that $(\mathcal{G}, A(n))$ is balanced, then $W(\mathcal{Q}) = W(\tilde{\mathcal{Q}})$. The Laplacian matrix of $(\mathcal{G}, A(n))$ is defined as

$$\mathcal{L}(n) = \begin{pmatrix} \sum\limits_{j \neq 1} a_{1j}(n) & -a_{12}(n) & \cdots & -a_{1l}(n) \\ -a_{21}(n) & \sum\limits_{j \neq 2} a_{2j}(n) & \cdots & -a_{2l}(n) \\ \vdots & \vdots & \ddots & \vdots \\ -a_{l1}(n) & -a_{l2}(n) & \cdots & \sum\limits_{j \neq l} a_{lj}(n) \end{pmatrix}.$$

The following result is standard in graph theory, and customarily called Kirchhoff's matrix tree theorem [19].

Lemma 1. *[19] Assume that $l \geq 2$. Let $c_i(n)$ denote the cofactor of the ith diagonal element of $\mathcal{L}(n)$, for any $n \in \mathbb{N}$. Then*

$$c_i(n) = \sum_{\mathcal{T} \in \mathbb{T}_i} W(\mathcal{T}), \quad i \in \mathbb{L},$$